Florian Cajori

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Содержание

Florian Cajori
William Oughtred / A great Seventeenth-Century Teacher of Mathematics

INTRODUCTION

In the year 1660 the Royal Society was founded by royal favor in London, although in reality its inception took place in 1645 when the Philosophical Society (or, as Boyle called it, the "Invisible College") came into being, which held meetings at Gresham College in London and later in Oxford. It was during the second half of the seventeenth century that Sir Isaac Newton, surrounded by a group of great men – Wallis, Hooke, Barrow, Halley, Cotes – carried on his epoch-making researches in mathematics, astronomy, and physics. But it is not this half-century of science in England, nor any of its great men, that especially engage our attention in this monograph. It is rather the half-century preceding, an epoch of preparation, when in the early times of the House of Stuart the sciences began to flourish in England. Says Dr. A. E. Shipley: "Whatever were the political and moral deficiencies of the Stuart kings, no one of them lacked intelligence in things artistic and scientific." It was at this time that mathematics, and particularly algebra, began to be cultivated with greater zeal, when elementary algebra with its symbolism as we know it now began to take its shape.

Biographers of Sir Isaac Newton make particular mention of five mathematical books which he read while a young student at Cambridge, namely, Euclid's *Elements*, Descartes's *Géométrie*, Vieta's *Works*, Van Schooten's *Miscellanies*, and Oughtred's *Clavis mathematicae*. The last of these books has been receiving increasing attention from the historians of algebra in recent years. We have prepared this sketch because we felt that there were points of interest in the life and activity of Oughtred which have not received adequate treatment. Historians have discussed his share in the development of symbolic algebra, but some have fallen into errors, due to inability to examine the original editions of Oughtred's *Clavis mathematicae*, which are quite rare and inaccessible to most readers. Moreover, historians have failed utterly to recognize his inventions of mathematical instruments, particularly the slide rule; they have completely overlooked his educational views and his ideas on mathematical teaching. The modern reader may pause with profit to consider briefly the career of this interesting man.

Oughtred was not a professional mathematician. He did not make his livelihood as a teacher of mathematics or as a writer, nor as an engineer who applies mathematics to the control and use of nature's forces. Oughtred was by profession a minister of the gospel. With him the study of mathematics was a side issue, a pleasure, a recreation. Like the great French algebraist, Vieta, from whom he drew much of his inspiration, he was an amateur mathematician. The word "amateur" must not be taken here in the sense of superficial or unthorough. Great Britain has had many men distinguished in science who pursued science as amateurs. Of such men Oughtred is one of the very earliest.

F. C.

CHAPTER I
OUGHTRED'S LIFE

AT SCHOOL AND UNIVERSITY

William Oughtred, or, as he sometimes wrote his name, *Owtred*, was born at Eton, the seat of Eton College, the year of his birth being variously given as 1573, 1574, and 1575. "His father," says Aubrey, "taught to write at Eaton, and was a scrivener; and understood common arithmetique, and 'twas no small helpe and furtherance to his son to be instructed in it when a schoole-boy."[1] He was a boy at Eton in the year of the Spanish Armada. At this famous school, which prepared boys for the universities, young Oughtred received thorough training in classical learning.

According to information received from F. L. Clarke, Bursar and Clerk of King's College, Cambridge, Oughtred was admitted at King's a scholar from Eton on September 1, 1592, at the age of seventeen. He was made Fellow at King's on September 1, 1595, while Elizabeth was still on the throne. He received in 1596 the degree of Bachelor of Arts and in 1600 that of Master of Arts. He vacated his fellowship about the beginning of August, 1603. His career at the University of Cambridge we present in his own words. He says:

> Next after Eaton schoole, I was bred up in Cambridge in Kings Colledge: of which society I was a member about eleven or twelve yeares: wherein how I behaved my selfe, going hand in hand with the rest of my ranke in the ordinary Academicall studies and exercises, and with what approbation, is well knowne and remembered by many: the time which over and above those usuall studies I employed upon the Mathematicall sciences, I redeemed night by night from my naturall sleep, defrauding my body, and inuring it to watching, cold, and labour, while most others tooke their rest. Neither did I therein seek only my private content, but the benefit of many: and by inciting, assisting, and instructing others, brought many into the love and study of those Arts, not only in our own, but in some other Colledges also: which some at this time (men far better than my selfe in learning, degree, and preferment) will most lovingly acknowledge.[2]

These words describe the struggles which every youth not endowed with the highest genius must make to achieve success. They show, moreover, the kindly feeling toward others and the delight he took throughout life in assisting anyone interested in mathematics. Oughtred's passion for this study is the more remarkable as neither at Eton nor at Cambridge did it receive emphasis. Even after his time at Cambridge mathematical studies and their applications were neglected there. Jeremiah Horrox was at Cambridge in 1633-35, desiring to make himself an astronomer.

> "But many impediments," says Horrox, "presented themselves: the tedious difficulty of the study itself deterred a mind not yet formed; the want of means oppressed, and still oppresses, the aspirations of my mind: but that which gave me most concern was that there was no one who could instruct me in the art, who could

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[1] Aubrey's *Brief Lives*, ed. A. Clark, Vol. II, Oxford, 1898, p. 106.

[2] "To the English Gentrie, and all others studious of the Mathematicks, which shall bee Readers hereof. The just Apologie of Wil: Ovghtred, against the slaunderous insimulations of Richard Delamain, in a Pamphlet called *Grammelogia*, or the Mathematicall Ring, or *Mirifica logarithmorum projectio circularis*" [1633?], p. 8. Hereafter we shall refer to this pamphlet as the *Apologeticall Epistle*, this name appearing on the page-headings.

> even help my endeavours by joining me in the study; such was the sloth and languor which had seized all… I found that books must be used instead of teachers."[3]

Some attention was given to Greek mathematicians, but the works of Italian, German, and French algebraists of the latter part of the sixteenth and beginning of the seventeenth century were quite unknown at Cambridge in Oughtred's day. It was part of his life-work as a mathematician to make algebra, as it was being developed in his time, accessible to English youths.

At the age of twenty-three Oughtred invented his *Easy Way of Delineating Sun-Dials by Geometry*, which, though not published until about half a century later, in the first English edition of Oughtred's *Clavis mathematicae* in 1647, was in the meantime translated into Latin by Christopher Wren, then a Gentleman Commoner of Wadham College, Oxford, now best known through his architectural creations. In 1600 Oughtred wrote a monograph on the construction of sun-dials upon a plane of any inclination, but that paper was withheld by him from publication until 1632. Sun-dials were interesting objects of study, since watches and pendulum clocks were then still unknown. All sorts of sun-dials, portable and non-portable, were used at that time and long afterward. Several of the college buildings at Oxford and Cambridge have sun-dials even at the present time.

[3] *Companion to the [British] Almanac of 1837*, p. 28, in an article by Augustus De Morgan on "Notices of English Mathematical and Astronomical Writers between the Norman Conquest and the Year 1600."

AS RECTOR AND AMATEUR MATHEMATICIAN

It was in 1604 that Oughtred entered upon his professional life-work as a preacher, being instituted to the vicarage of Shalford in Surrey. In 1610 he was made rector of Albury, where he spent the remainder of his long life. Since the era of the Reformation two of the rectors of Albury obtained great celebrity from their varied talents and acquirements – our William Oughtred and Samuel Horsley. Oughtred continued to devote his spare time to mathematics, as he had done in college. A great mathematical invention made by a Scotchman soon commanded his attention – the invention of logarithms. An informant writes as follows:

> Lord Napier, in 1614, published at Edinburgh his *Mirifici logarithmorum canonis descriptio*… It presently fell into the hands of Mr. Briggs, then geometry-reader at Gresham College in London: and that gentleman, forming a design to perfect Lord Napier's plan, consulted Oughtred upon it; who probably wrote his *Treatise of Trigonometry* about the same time, since it is evidently formed upon the plan of Lord Napier's *Canon*.[4]

It will be shown later that Oughtred is very probably the author of an "Appendix" which appeared in the 1618 edition of Edward Wright's translation into English of John Napier's *Descriptio*. This "Appendix" relates to logarithms and is an able document, containing several points of historical interest. Mr. Arthur Hutchinson of Pembroke College informs me that in the university library at Cambridge there is a copy of Napier's *Constructio* (1619) bound up with a copy of Kepler's *Chilias logarithmorum* (1624), that at the beginning of the *Constructio* is a blank leaf, and before this occurs the title-page only of Napier's *Descriptio* (1619), at the top of which appears Oughtred's autograph. The history of this interesting signature is unknown.

[4] *New and General Biographical Dictionary* (John Nichols), London, 1784, art. "Oughtred."

HIS WIFE

In 1606 he married Christ'sgift Caryll, daughter of Caryll, Esq., of Tangley, in an adjoining parish.[5] We know very little about Oughtred's family life. The records at King's College, Cambridge,[6] mention a son, but it is certain that there were more children. A daughter was married to Christopher Brookes. But there is no confirmation of Aubrey's statements,[7] according to which Oughtred had nine sons and four daughters. Reference to the wife and children is sometimes made in the correspondence with Oughtred. In 1616 J. Hales writes, "I pray let me be remembered, though unknown, to Mistress Oughtred."[8]

As we shall see later, Oughtred had a great many young men who came to his house and remained there free of charge to receive instruction in mathematics, which was likewise gratuitous. This being the case, certainly great appreciation was due to Mrs. Oughtred, upon whom the burden of hospitality must have fallen. Yet chroniclers are singularly silent in regard to her. Hers was evidently a life of obscurity and service. We greatly doubt the accuracy of the following item handed down by Aubrey; it cannot be a true characterization:

> His wife was a penurious woman, and would not allow him to burne candle after supper, by which meanes many a good notion is lost, and many a probleme unsolved; so that Mr. [Thomas] Henshawe, when he was there, bought candle, which was a great comfort to the old man.[9]

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[5] Rev. Owen Manning, *History of Antiquities in Surrey*, Vol. II, p. 132.

[6] *Skeleton Collegii Regalis Cantab.: Or A Catalogue of All the Provosts, Fellows and Scholars, of the King's College.. since the Foundation Thereof*, Vol. II, "William Oughtred."

[7] Aubrey, *op. cit.*, Vol. II, p. 107.

[8] Rigaud, *Correspondence of Scientific Men of the Seventeenth Century*, Oxford, Vol. I, 1841, p. 5.

[9] Aubrey, *op. cit.*, Vol. II, p. 110.

IN DANGER OF SEQUESTRATION

Oughtred spent his years in "unremitted attention to his favourite study," sometimes, it has been whispered, to the neglect of his rectorial duties. Says Aubrey:

> I have heard his neighbour ministers say that he was a pittiful preacher; the reason was because he never studyed it, but bent all his thoughts on the mathematiques; but when he was in danger of being sequestred for a royalist, he fell to the study of divinity, and preacht (they sayd) admirably well, even in his old age.[10]

This remark on sequestration brings to mind one of the political and religious struggles of the time, the episcopacy against the independent movements. Says Manning:

> In 1646 he was cited before the Committee for Ecclesiastical Affairs, where many articles had been deposed against him; but, by the favour of Sir *Bulstrode Whitlock* and others, who, at the intercession of *William Lilye* the Astrologer, appeared in great numbers on his behalf, he had a majority on his side, and so escaped a sequestration.[11]

Not without interest is the account of this matter given by Lilly himself:

> About this Time, the most famous Mathematician of all Europe, (Mr. William Oughtred, Parson of Aldbury in Surrey) was in Danger of Sequestration by the Committee of or for plunder'd Ministers; (*Ambo-dexters* they were;) several inconsiderable Articles were deposed and sworn against him, material enough to have sequestred him, but that, upon his Day of hearing, I applied my self to Sir Bolstrode Whitlock, and all my own old Friends, who in such Numbers appeared in his Behalf, that though the Chairman and many other Presbyterian Members were stiff against him, yet he was cleared by the major Number. The truth is, he had a considerable Parsonage, and that only was enough to sequester any moderate Judgment: He was also well known to affect his Majesty [Charles I]. In these Times many worthy Ministers lost their Livings or Benefices, for not complying with the Three-penny Directory.[12]

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[10] *Ibid.*, p. 111.

[11] *Op. cit.*, Vol. II, p. 132.

[12] *Mr. William Lilly's History of His Life and Times, From the Year 1602 to 1681*, London, 1715, p. 58.

HIS TEACHING

Oughtred had few personal enemies. His pupils held him in highest esteem and showed deep gratitude; only one pupil must be excepted, Richard Delamain. Against him arose a bitter controversy which saddened the life of Oughtred, then an old man. It involved, as we shall see later, the priority of invention of the circular slide rule and of a horizontal instrument or portable sun-dial. In defense of himself, Oughtred wrote in 1633 or 1634 the *Apologeticall Epistle*, from which we quoted above. This document contains biographical details, in part as follows:

> Ever since my departure from the Vniversity, which is about thirty yeares, I have lived neere to the Towne of Guildford in Surrey: where, whether *I have taken so much liberty to the losse of time, and the neglect of my calling* the whole Countrey thereabout, both Gentry and others, to whom I am full well knowne, will quickely informe him; my house being not past three and twenty miles from London: and yet I so hid my selve at home, that I seldomly travelled so farre as London once in a yeare. Indeed the life and mind of man cannot endure without some interchangeablenesse of recreation, and pawses from the intensive actions of our severall callings; and every man is drawne with his owne delight. My recreations have been diversity of studies: and as oft as I was toyled with the labour of my owne profession, I have allayed that tediousnesse by walking in the pleasant and more then Elysian fields of the diverse and various parts of humane learning, and not the Mathematics onely.

Even the opponents of Delamain must be grateful to him for having been the means of drawing from Oughtred such interesting biographical details. Oughtred proceeds to tell how, about 1628, he was induced to write his *Clavis mathematicae*, upon which his reputation as a mathematician largely rests:

> About five yeares since, the Earle of Arundell my most honourable Lord in a time of his private retiring to his house in the countrey then at West Horsley, foure small miles from me (though since he hath a house in Aldebury the parish where I live) hearing of me (by what meanes I know not) was pleased to send for me: and afterward at London to appoint mee a Chamber of his owne house: where, at such times, and in such manner as it seemed him good to imploy me, and when I might not inconveniently be spared from my charge, I have been most ready to present my selfe in all humble and affectionate service: I hope also without the offence of God, the transgression of the good Lawes of this Land, neglect of my calling, or the deserved scandall of any good man…
>
> And although I am no *mercenary man*, nor make profession to teach any one in these arts for gaine and recompence, but as I serve at the Altar, so I live onely of the Altar: yet in those interims that I am at London in my Lords service, I have been still much frequented both by Natives and Strangers, for my resolution and instruction in many difficult poynts of Art; and have most freely and lovingly imparted my selfe and my skill, such as I had, to their contentments, and much honourable acknowledgement of their obligation to my Lord for bringing mee to London, hath beene testifyed by many. Of which my liberallity and unwearyed readinesse to doe good to all, scarce any one can give more ample testimony then R. D. himselfe can: would he be but pleased to allay the shame of this his hot and eager contention, blowne up onely with the full bellowes of intended glory and gaine;.. they [the subjects in which Delamain received assistance from Oughtred] were the first elements of Astronomie concerning the second motions of the fixed starres,

> and of the Sunne and Moone; they were the first elements of Conics, to delineate those sections: they were the first elements of Optics, Catoptrics, and Dioptrics: of all which you knew nothing at all.

These last passages are instructive as showing what topics were taken up for study with some of his pupils. The chief subject of interest with most of them was algebra, which at that time was just beginning to draw the attention of English lovers of mathematics.

Oughtred carried on an extensive correspondence on mathematical subjects. He was frequently called upon to assist in the solution of knotty problems – sometimes to his annoyance, perhaps, as is shown by the following letter which he wrote in 1642 to a stranger, named Price:

> It is true that I have bestowed such vacant time, as I could gain from the study of divinity, (which is my calling,) upon human knowledges, and, amongst other, upon the mathematics, wherein the little skill I have attained, being compared with others of my profession, who for the most part contenting themselves only with their own way, refuse to tread these salebrous and uneasy paths, may peradventure seem the more. But now being in years and mindful of mine end, and having paid dearly for my former delights both in my health and state, besides the prejudice of such, who not considering what incessant labour may produce, reckon so much wanting unto me in my proper calling, as they think I have acquired in other sciences; by which opinion (not of the vulgar only) I have suffered both disrespect, and also hinderance in some small perferments I have aimed at. I have therefore now learned to spare myself, and am not willing to descend again in arenam, and to serve such ungrateful muses. Yet, sir, at your request I have perused your problem… Your problem is easily wrought per Nicomedis conchoidem lineam.[13]

[13] Rigaud, *op. cit.*, Vol. I, p. 60.

APPEARANCE AND HABITS

Aubrey gives information about the appearance and habits of Oughtred:

> He was a little man, had black haire, and blacke eies (with a great deal of spirit). His head was always working. He would drawe lines and diagrams on the dust…
>
> He [his oldest son Benjamin] told me that his father did use to lye a bed till eleaven or twelve a clock, with his doublet on, ever since he can remember. Studyed late at night; went not to bed till 11 a clock; had his tinder box by him; and on the top of his bed-staffe, he had his inke-horne fix't. He slept but little. Sometimes he went not to bed in two or three nights, and would not come downe to meales till he had found out the *quaesitum*.
>
> He was more famous abroad for his learning, and more esteemed, then at home. Severall great mathematicians came over into England on purpose to converse with him. His countrey neighbours (though they understood not his worth) knew that there must be extraordinary worth in him, that he was so visited by foreigners…
>
> When learned foreigners came and sawe how privately he lived, they did admire and blesse themselves, that a person of so much worth and learning should not be better provided for…
>
> He has told bishop Ward, and Mr. Elias Ashmole (who was his neighbour), that "on this spott of ground" (or "leaning against this oake" or "that ashe"), "the solution of such or such a probleme came into my head, as if infused by a divine genius, after I had thought on it without successe for a yeare, two, or three."..
>
> Nicolaus Mercator, Holsatus.. went to see him few yeares before he dyed…
>
> The right hon[ble] Thomas Howard, earle of Arundel and Surrey, Lord High Marshall of England, was his great patron, and loved him intirely. One time they were like to have been killed together by the fall at Albury of a grott, which fell downe but just as they were come out.[14]

Oughtred's friends convey the impression that, in the main, Oughtred enjoyed a comfortable living at Albury. Only once appear indications of financial embarrassment. About 1634 one of his pupils, W. Robinson, writes as follows:

> I protest unto you sincerely, were I as able as some, at whose hands you have merited exceedingly, or (to speak more absolutely) as able as willing, I would as freely give you 500 *l.* per ann. as 500 pence; and I cannot but be astonished at this our age, wherein pelf and dross is made their summum bonum, and the best part of man, with the true ornaments thereof, science and knowledge, are so slighted…[15]

In his letters Oughtred complains several times of the limitations for work and the infirmities due to his advancing old age. The impression he made upon others was quite different. Says one biographer:

> He sometimes amused himself with archery, and sometimes practised as a surveyor of land… He was sprightly and active, when more than eighty years of age.[16]

[14] Aubrey, *op. cit.*, Vol. II, p. 107.
[15] Rigaud, *op. cit.*, Vol. I, p. 16.
[16] Owen Manning, *op. cit.*, p. 132.

Another informant says that Oughtred was

> as facetious in Greek and Latine as solid in Arithmetique, Astronomy, and the sphere of all Measures, Musick, etc.; exact in his style as in his judgment; handling his Cube, and other Instruments at eighty, as steadily, as others did at thirty; owing this, he said, to temperance and Archery; principling his people with plain and solid truths, as he did the world with great and useful Arts; advancing new Inventions in all things but Religion. Which in its old order and decency he maintained secure in his privacy, prudence, meekness, simplicity, resolution, patience, and contentment.[17]

[17] *New and General Biographical Dictionary* (John Nichols), London, 1784, art. "Oughtred."

ALLEGED TRAVEL ABROAD

According to certain sources of information, Oughtred traveled on the European Continent and was invited to change his abode to the Continent. We have seen no statement from Oughtred himself on this matter. He seldom referred to himself in his books and letters. The autobiography contained in his *Apologeticall Epistle* was written a quarter of a century before his death. Aubrey gives the following:

> In the time of the civill warres the duke of Florence invited him over, and offered him 500 li. per annum; but he would not accept it, because of his religion.[18]

A portrait of Oughtred, painted in 1646 by Hollar and inserted in the English edition of the *Clavis* of 1647, contains underneath the following lines:

> "Haec est Oughtredi senio labantis imago
> Itala quam cupiit, Terra Britanna tulit."

In the sketch of Oughtred by Owen Manning it is confessed that "it is not known to what this alludes; but possibly he might have been in *Italy* with his patron, the Earl of Arundel."[19] It would seem quite certain either that Oughtred traveled in Europe or that he received some sort of an offer to settle in Italy. In view of Aubrey's explicit statement and of Oughtred's well-known habit of confining himself to his duties and studies in his own parish, seldom going even as far as London, we strongly incline to the opinion that he did not travel on the Continent, but that he received an offer from some patron of the sciences – possibly some distinguished visitor – to settle in Italy.

[18] *Op. cit.*, Vol. II, p. 110.
[19] Rev. Owen Manning, *The History and Antiquities of Surrey*, Vol. II, London, 1809, p. 132.

HIS DEATH

He died at Albury, June 30, 1660, aged about eighty-six years. Of his last days and death, Aubrey speaks as follows:

> Before he dyed he burned a world of papers, and sayd that the world was not worthy of them; he was so superb. He burned also severall printed bookes, and would not stirre, till they were consumed… I myselfe have his Pitiscus, imbelished with his excellent marginall notes, which I esteeme as a great rarity. I wish I could also have got his Bilingsley's Euclid, which John Collins sayes was full of his annotations…
>
> Ralph Greatrex, his great friend, the mathematicall instrument-maker, sayed he conceived he dyed with joy for the comeing-in of the king, which was the 29th of May before. "And are yee sure he is restored?" – "Then give me a glasse of sack to drinke his sacred majestie's health." His spirits were then quite upon the wing to fly away…[20]

In this passage, as in others, due allowance must be made for Aubrey's lack of discrimination. He was not in the habit of sifting facts from mere gossip. That Oughtred should have declared that the world was not worthy of his papers or manuscripts is not in consonance with the sweetness of disposition ordinarily attributed to him. More probable was the feeling that the papers he burned – possibly old sermons – were of no particular value to the world. That he did not destroy a large mass of mathematical manuscripts is evident from the fact that a considerable number of them came after his death into the hands of Sir Charles Scarborough, M.D., under whose supervision some of them were carefully revised and published at Oxford in 1677 under the title of *Opuscula mathematica hactenus inedita*.

Aubrey's story of Oughtred's mode of death has been as widely circulated in every modern biographical sketch as has his slander of Mrs. Oughtred by claiming that she was so penurious that she would deny him the use of candles to read by. Oughtred died on June 30; the Restoration occurred on May 29. No doubt Oughtred rejoiced over the Restoration, but the story of his drinking "a glass of sack" to his Majesty's health, and then dying of joy is surely apocryphal. De Morgan humorously remarks, "It should be added, by way of excuse, that he was eighty-six years old."[21]

[20] *Op. cit.*, Vol. II, 1898, p. 111.

[21] *Budget of Paradoxes*, London, 1872, p. 451; 2d ed., Chicago and London, 1915, Vol. II, p. 303.

CHAPTER II
PRINCIPAL WORKS

"CLAVIS MATHEMATICAE"

Passing to the consideration of Oughtred's mathematical books, we begin with the observation that he showed a marked disinclination to give his writings to the press. His first paper on sun-dials was written at the age of twenty-three, but we are not aware that more than one brief mathematical manuscript was printed before his fifty-seventh year. In every instance, publication in printed form seems to have been due to pressure exerted by one or more of his patrons, pupils, or friends. Some of his manuscripts were lent out to his pupils, who prepared copies for their own use. In some instances they urged upon him the desirability of publication and assisted in preparing copy for the printer. The earliest and best-known book of Oughtred was his *Clavis mathematicae*, to which repeated allusion has already been made. As he himself informs us, he was employed by the Earl of Arundel about 1628 to instruct the Earl's son, Lord William Howard (afterward Viscount Stafford) in the mathematics. For the use of this young man Oughtred composed a treatise on algebra which was published in Latin in the year 1631 at the urgent request of a kinsman of the young man, Charles Cavendish, a patron of learning.

The *Clavis mathematicae*,[22] in its first edition of 1631, was a booklet of only 88 small pages. Yet it contained in very condensed form the essentials of arithmetic and algebra as known at that time.

Aside from the addition of four tracts, the 1631 edition underwent some changes in the editions of 1647 and 1648, which two are much alike. The twenty chapters of 1631 are reduced to nineteen in 1647 and in all the later editions. Numerous minute alterations from the 1631 edition occur in all parts of the books of 1647 and 1648. The material of the last three chapters of the 1631 edition is rearranged, with some slight additions here and there. The 1648 edition has no preface. In the print of 1652 there are only slight alterations from the 1648 edition; after that the book underwent hardly any changes, except for the number of tracts appended, and brief explanatory notes added at the close of the chapters in the English editions of 1694 and 1702. The 1652 and 1667 editions were seen through the press by John Wallis; the 1698 impression contains on the title-page the words: *Ex Recognitione D. Johannis Wallis, S.T.D. Geometriae Professoris Saviliani*.

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[22] The full title of the *Clavis* of 1631 is as follows: *Arithmeticae in numeris et speciebvs institvtio: Qvae tvm logisticae, tvm analyticae, atqve adeo totivs mathematicae, qvasi clavis est. – Ad nobilissimvm spectatissimumque invenem Dn. Gvilelmvm Howard, Ordinis qui dicitur, Balnei Equitem, honoratissimi Dn. Thomae, Comitis Arvndeliae & Svrriae, Comitis Mareschalli Angliae, &c filium. – Londini, Apud Thomam Harpervm.* M.DC.XXXI.In all there appeared five Latin editions, the second in 1648 at London, the third in 1652 at Oxford, the fourth in 1667 at Oxford, the fifth in 1693 and 1698 at Oxford. There were two independent English editions: the first in 1647 at London, translated in greater part by Robert Wood of Lincoln College, Oxford, as is stated in the preface to the 1652 Latin edition; the second in 1694 and 1702 is a new translation, the preface being written and the book recommended by the astronomer Edmund Halley. The 1694 and 1702 impressions labored under the defect of many sense-disturbing errors due to careless reading of the proofs. All the editions of the *Clavis*, after the first edition, had one or more of the following tracts added on:*Eq.=De Aequationum affectarvm resolvtione in numeris.Eu.=Elementi decimi Euclidis declaratio.So.=De Solidis regularibus, tractatus.An.=De Anatocismo, sive usura composita.Fa.=Regula falsae positionis.Ar.=Theorematum in libris Archimedis de Sphaera & cylindro declaratio.Ho.=Horologia scioterica in plano, geometricè delineandi modus.*The abbreviated titles given here are, of course, our own. The lists of tracts added to the *Clavis mathematicae* of 1631 in its later editions, given in the order in which the tracts appear in each edition, are as follows: *Clavis* of 1647, *Eq.*, *An.*, *Fa.*, *Ho.*; *Clavis* of 1648, *Eq.*, *An.*, *Fa.*, *Eu.*, *So.*; *Clavis* of 1652, *Eq.*, *Eu.*, *So.*, *An.*, *Fa.*, *Ar.*, *Ho.*; *Clavis* of 1667, *Eq.*, *Eu.*, *So.*, *An.*, *Fa.*, *Ar.*, *Ho.*; *Clavis* of 1693 and 1698, *Eq.*, *Eu.*, *So.*, *An.*, *Fa.*, *Ar.*, *Ho.*; *Clavis* of 1694 and 1702, *Eq.*The title-page of the *Clavis* was considerably modified after the first edition. Thus, the 1652 Latin edition has this title-page: *Guilelmi Oughtred Aetonensis, quondam Collegii Regalis in Cantabrigia Socii, Clavis mathematicae denvo limata, sive potius fabricata. Cum aliis quibusdam ejusdem commentationibus, quae in sequenti pagina recensentur. Editio tertia auctior & emendatior. Oxoniae, Excudebat Leon. Lichfield, Veneunt apud Tho. Robinson. 1652.*

The cost of publishing may be a matter of some interest. When arranging for the printing of the 1667 edition of the *Clavis*, Wallis wrote Collins: "I told you in my last what price she [Mrs. Lichfield] expects for it, as I have formerly understood from her, viz., £ 40 for the impression, which is about 9½*d.* a book."[23]

As compared with other contemporary works on algebra, Oughtred's distinguishes itself for the amount of symbolism used, particularly in the treatment of geometric problems. Extraordinary emphasis was placed upon what he called in the *Clavis* the "analytical art."[24] By that term he did not mean our modern analysis or analytical geometry, but the art "in which by taking the thing sought as knowne, we finde out that we seeke."[25] He meant to express by it condensed processes of rigid, logical deduction expressed by appropriate symbols, as contrasted with mere description or elucidation by passages fraught with verbosity. In the preface to the first edition (1631) he says:

> In this little book I make known.. the rules relating to fundamentals, collected together, just like a bundle, and adapted to the explanation of as many problems as possible.

As stated in this preface, one of his reasons for publishing the book, is

> … that like Ariadne I might offer a thread to mathematical study by which the mysteries of this science might be revealed, and direction given to the best authors of antiquity, Euclid, Archimedes, the great geometrician Apollonius of Perga, and others, so as to be easily and thoroughly understood, their theorems being added, not only because to many they are the height and depth of mathematical science (I ignore the would-be mathematicians who occupy themselves only with the so-called practice, which is in reality mere juggler's tricks with instruments, the surface so to speak, pursued with a disregard of the great art, a contemptible picture), but also to show with what keenness they have penetrated, with what mass of equations, comparisons, reductions, conversions and disquisitions these heroes have ornamented, increased and invented this most beautiful science.

The *Clavis* opens with an explanation of the Hindu-Arabic notation and of decimal fractions. Noteworthy is the absence of the words "million," "billion," etc. Though used on the Continent by certain mathematical writers long before this, these words did not become current in English mathematical books until the eighteenth century. The author was a great admirer of decimal fractions, but failed to introduce the notation which in later centuries came to be universally adopted. Oughtred wrote 0.56 in this manner 0|56; the point he used to designate ratio. Thus 3:4 was written by him 3·4. The decimal point (or comma) was first used by the inventor of logarithms, John Napier, as early as 1616 and 1617. Although Oughtred had mastered the theory of logarithms soon after their publication in 1614 and was a great admirer of Napier, he preferred to use the dot for the designation of *ratio*. This notation of ratio is used in all his mathematical books, except in two instances. The two dots (:) occur as symbols of ratio in some parts of Oughtred's posthumous work, *Opuscula mathematica hactenus inedita*, Oxford, 1677, but may have been due to the editors and not to Oughtred himself. Then again the two dots (:) are used to designate ratio on the last two pages of the tables of the Latin edition of Oughtred's *Trigonometria* of 1657. In all other parts of that book the dot (·) is used. Probably someone who supervised the printing of the tables introduced the (:) on the last two pages, following the logarithmic tables, where methods of interpolation are explained. The probability of this conjecture is the stronger, because in the English edition of the *Trigonometrie*, brought out the

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[23] Rigaud, *op. cit.*, Vol. II, p. 476.

[24] See, for instance, the *Clavis mathematicae* of 1652, where he expresses himself thus (p. 4): "Speciosa haec Arithmetica arti Analyticae (per quam ex sumptione quaesiti, tanquam noti, investigatur quaesitum) multo accommodatior est, quam illa numerosa."

[25] Oughtred, *The Key of the Mathematicks*, London, 1647, p. 4.

same year (1657) but *after* the Latin edition, the notation (:) at the end of the book is replaced by the usual (·), except that in some copies of the English edition the explanations at the end are omitted altogether.

Oughtred introduces an interesting, and at the same time new, feature of an abbreviated multiplication and an abbreviated division of decimal fractions. On this point he took a position far in advance of his time. The part on abbreviated multiplication was rewritten in slightly enlarged form and with some unimportant alterations in the later edition of the *Clavis*. We give it as it occurs in the revision. Four cases are given. In finding the product of 246|914 and 35|27, "if you would have the Product without any Parts" (without any decimal part), "set the place of Unity of the lesser under the place of Unity in the greater: as in the Example," writing the figures of the lesser number in *inverse order*. From the example it will be seen that he begins by multiplying by 3, the right-hand digit of the multiplier. In the first edition of the *Clavis* he began with 7, the left digit. Observe also that he "carries" the nearest tens in the product of each lower digit and the upper digit one place to its right. For instance, he takes 7×4=28 and carries 3, then he finds 7×2+3=17 and writes down 17.

```
2 4 6|9 1 4  7 2|5 3
-------
7 4 0 7
1 2 3 5
    4 9
    1 7
-------
8 7 0 8
```

The second case supposes that "you would have the Product with some places of parts" (decimals), say 4: "Set the place of Unity of the lesser Number under the Fourth place of the Parts of the greater." The multiplication of 246|914 by 35|27 is now performed thus:

```
2 4 6|9 1 4          7 2|5 3
---------------
7 4 0 7 4 2 0 0
1 2 3 4 5 7 0 0
    4 9 3 8 2 8
    1 7 2 8 4 0
---------------
8 7 0 8|6 5 6 8
```

In the third and fourth cases are considered factors which appear as integers, but are in reality decimals; for instance, the sine of 54° is given in the tables as 80902 when in reality it is .80902.

Of interest as regards the use of the word "parabola" is the following: "The Number found by Division is called the *Quotient*, or also *Parabola*, because it arises out of the Application of a plain Number to a given Longitude, that a congruous Latitude may be found."[26] This is in harmony with etymological dictionaries which speak of a parabola as the application of a given area to a given straight line. The dividend or product is the area; the divisor or factor is the line.

Oughtred gives two processes of long division. The first is identical with the modern process, except that the divisor is written below every remainder, each digit of the divisor being crossed out as soon as it has been used in the partial multiplication. The second method of long division is one of the several types of the old "scratch method." This antiquated process held its place by the side of the modern method in all editions of the *Clavis*. The author divides 467023 by 357|0926425, giving the following instructions: "Take as many of the first Figures of the Divisor as are necessary, for the first Divisor, and then in every following particular Division drop one of the Figures of the Divisor towards the Left Hand, till you have got a competent Quotient." He does not explain abbreviated division as thoroughly as abbreviated multiplication.

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[26] *Clavis* 1694, p. 19, and the *Clavis* of 1631, p. 8.

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                      17
                     3/0/3/
2/8/0/3/                   1/0/9/9/3/0/  357|0926425)
4/6/7/0/2/3/  (1307|80
3/5/7/0/9/3/                   1/0/7/1/2/7/
2/5/0/0/                      2/8/6/
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Oughtred does not examine the degree of reliability or accuracy of his processes of abbreviated multiplication and division. Here as in other places he gives in condensed statement the mode of procedure, without further discussion.

He does not attempt to establish the rules for the addition, subtraction, multiplication, and division of positive and negative numbers. "If the Signs are both alike, the Product will be affirmative, if unlike, negative"; then he proceeds to applications. This attitude is superior to that of many writers of the eighteenth and nineteenth centuries, on pedagogical as well as logical grounds: pedagogically, because the beginner in the study of algebra is not in a position to appreciate an abstract train of thought, as every teacher well knows, and derives better intellectual exercise from the applications of the rules to problems; logically, because the rule of signs in multiplication does not admit of rigorous proof, unless some other assumption is first made which is no less arbitrary than the rule itself. It is well known that the proofs of the rule of signs given by eighteenth-century writers are invalid. Somewhere they involve some surreptitious assumption. This criticism applies even to the proof given by Laplace, which tacitly assumes the distributive law in multiplication.

A word should be said on Oughtred's definition of + and – . He recognizes their double function in algebra by saying (*Clavis*, 1631, p. 2): "Signum additionis, sive affirmationis, est + plus" and "Signum subductionis, sive negationis est – minus." They are symbols which indicate the *quality* of numbers in some instances and *operations* of addition or subtraction in other instances. In the 1694 edition of the *Clavis*, thirty-four years after the death of Oughtred, these symbols are defined as signifying operations only, but are actually used to signify the quality of numbers as well. In this respect the 1694 edition marks a recrudescence.

The characteristic in the *Clavis* that is most striking to a modern reader is the total absence of indexes or exponents. There is much discussion in the leading treatises of the latter part of the sixteenth and the early part of the seventeenth century on the theory of indexes, but the modern exponential notation, a^n, is of later date. The modern notation, for positive integral exponents, first appears in Descartes' *Géométrie*, 1637; fractional and negative exponents were first used in the modern form by Sir Isaac Newton, in his announcement of the binomial formula, in a letter written in 1676. This total absence of our modern exponential notation in Oughtred's *Clavis* gives it a strange aspect. Like Vieta, Oughtred uses ordinarily the capital letters, *A*, *B*, *C*… to designate given numbers; A^2 is written *Aq*, A^3 is written *Ac*; for A^4, A^5, A^6 he has, respectively, *Aqq*, *Aqc*, *Acc*. Only on rare occasions, usually when some parallelism in notation is aimed at, does he use small letters[27] to represent numbers or magnitudes. Powers of binomials or polynomials are marked by prefixing the capital letters *Q* (for square), *C* (for cube), *QQ* (for the fourth power), *QC* (for the fifth power), etc.

Oughtred does not express aggregation by (). Parentheses had been used by Girard, and by Clavius as early as 1609,[28] but did not come into general use in mathematical language until the time of Leibniz and the Bernoullis. Oughtred indicates aggregation by writing a colon (:) at both ends. Thus, *Q*:*A*-*E*: means with him $(A\text{-}E)^2$. Similarly, √*q*:*A*+*E*: means $\sqrt{(A+E)}$. The two dots at the end are frequently omitted when the part affected includes all the terms of the polynomial to the end. Thus, *C*:*A*+*B*-*E*=.. means $(A+B\text{-}E)^3$=.. There are still further departures from this notation, but they occur so seldom that we incline to the interpretation that they are simply printer's errors. For proportion

[27] See for instance, Oughtred's *Elementi decimi Euclidis declaratio*, 1652, p. 1, where he uses *A* and *E*, and also *a* and *e*.

[28] See *Christophori Clavii Bambergensis Operum mathematicorum, tomus secundus*, Moguntiae, M.DC.XI, algebra, p. 39.

Oughtred uses the symbol (::). The proportion $a{:}b{=}c{:}d$ appears in his notation $a{\cdot}b{::}c{\cdot}d$. Apparently, a proportion was not fully recognized in this day as being the expression of an equality of ratios. That probably explains why he did not use = here as in the notation of ordinary equations. Yet Oughtred must have been very close to the interpretation of a proportion as an equality; for he says in his *Elementi decimi Euclidis declaratio*

Конец ознакомительного фрагмента.