PETRUS RAMUS

THE WAY TO GEOMETRY

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Содержание

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TO THE
WORSHIPFVL

M. Iohn Greaves, *Professor* of

***Geometry* in *Gresham Colledge* London;**

All happinesse

SIR,

Your acquaintance with the Author before his death was not long, which I have oft heard you say, you counted your great unhappinesse, but within a short time after, you knew not well whether to count your selfe more happie in that you once knew him, or unhappy in that upon your acquaintance you so suddenly lost him. This his worke then being to come forth to the censorious

eye of the world, and as the manner usually is to have some Patronage, I have thought good to dedicate it to your selfe; and that for these two reasons especially.

First, in respect of the sympathy betwixt it, and your studies; Laboures of this nature being usually offered to such persons whose profession is that way setled.

Secondly, for the great love and respect you alwayes shewed to the Author, being indeed a man that would deserve no lesse, humble, void of pride, ever ready to impart his knowledge to others in what kind soever, loving and affecting those that affected learning.

For these respects then, I offer to you this Worke of your so much honoured friend. I my selfe also (as it is no lesse my duty) for his sake striving to make you hereby some part of a requitall, least I should be found guilty of ingratitude, which is a solecisme in manners, if having so fit an opportunity, I should not expresse to the world some Testimonie of love to you, who so much loved him. I desire then (good Sir) your kind acceptance of it, you knowing so well the ability of the Author, and being also able to judge of a Worke of this nature, and in that respect the better able to defend it from the furie of envious Detractours, of which there are not few. Thus with my best wishes to you, as to my much respected friend, I rest.

Yours to be commanded in

any thing that he is able.
Iohn Clerke.

To the Reader

Friendly Reader, that which is here set forth to thy view, is a Translation out of *Ramus*. Formerly indeed Translated by one M^r^. *Thomas Hood*, but never before set forth with the Demonstrations and Diagrammes, which being cut before the Authors death, and the Worke it selfe finished, the Coppie I having in mine hands, never had thought for the promulgation of it, but that it should have died with its Author, considering no small prejudice usually attends the printing of dead mens Workes, and wee see the times, the world is now all eare and tongue, the most given with the *Athenians*, to little else than to heare and tell newes: And if *Apelles* that skilfull Artist alwayes found somewhat to be amended in those Pictures which he had most curiously drawne; surely much in this Worke might have beene amended if the Authour had lived to refine it, but in that it was onely the first draught, and that he was prevented by death of a second view, though perused by others before the Presse; I was ever unwilling to the publication, but that I was often and much solicited with iteration of strong importunity, and so in the end over-ruled: perswading me from time to time unto it, and that it being finished by the Authour, it was farre better

to be published, though with some errours and escapes, than to be onely moths-meat, and so utterly lost. I would have thee, Courteous Reader know, that it is no conceit of the worth of the thing that I should expose the name and credit of the Authour to a publike censure; yet I durst be bold to say, had he lived to have fitted it, and corrected the Presse, the worke would have pointed out the workeman. For I may say, without vaine ostentation, he was a man of worth and note, and there was not that kinde of learning in which he had not some knowledge, but especially for the Easterne tongues, those deepe and profound Studies, in the judgement of the learned, which knew him well, he hath not left his fellow behind him; as his Workes also in Manuscript now extant in the publike Library of the famous Vniversity of Cambridge; do testifie no lesse; for him then being so grave and learned a Divine to meddle with a worke of this nature, he gives thee a reason in his owne following Preface for his principall end and intent of taking this Worke in hand, was not for the deepe and Iudiciall, but for the shallowest skull, the good and profit of the simpler sort, who as it was in the Latine, were able to get little or no benifite from it. Therefore considering the worth of the Authour, and his intent in the Worke. Reade it favourably, and if the faults be not too great, cover them with the mantle of love, and judge charitably offences unwillingly committed, and doe according to the termes of equitie, as thou wouldest be done unto, but it is a common saying, as *Printers* get Copies for their profit, so Readers often buy and reade for their pleasure; and

there is no worke so exactly done that can escape the malevolous disposition of some detracting spirits, to whom I say, as one well, *Facilius est unicuivis nostrum aliena curiosè observare: quam proproia negotia rectè agere*. It is a great deale more easie to carpe at other mens doings, than to give better of his owne. And as *Arist.* τό πάσιν ἀρέσαι δυσχερέστατόν ἐστι; *omnibus placere difficilimum est.* But wherefore, Gentle Reader, should I make any doubt of thy curtesie, and favourable acceptance; for surely there can be nothing more contrary to equitie, than to speake evill of those that have taken paines to doe good, a Pagan would hardly doe this, much lesse I hope any good Christian. Read then, and if by reading, thou reapest any profit, I have my desire, if not, the fault shall be thine owne, reading haply more to judge and censure, than for any good and benefit which otherwise may be received from it; let but the same mind towards thine owne good possesse thee in reading it, as did the Author in writing it, and there shall be no neede to doubt of thy profit by it.

Thine in the common
bond of love,
Iohn Clerke.

The Authors Preface

Two things, I feare me, will here be objected against me: The one concerneth my selfe, directly: The other mine Author, and the worke I have taken in hand the translating of him. Concerning my selfe, I suppose, some will aske, Why I being a Divine; should meddle or busie my selfe with these prophane studies? Geometry *may no way further Divinity, and therefore is no fit study for a Divine? This objection seemeth to smell of Brownisme, that is, of a ranke peevish humour overflowing the stomach of some, whereby they are caused to loath all manner of solid learning, yea of true Divinity it selfe, and therefore it doth not deserve an answer: And this we in our Title before signified. For we have not taken this paines for Turkes and others, who by the lawes of their profession are bound to abandon all manner of learning. But if any man shall propose it, as a question, with a desire of satisfaction, we are ready to answer him to the best of our abilitie. First, that* Theologia vera est ars artium & scientia scientiarum, *Divinity is the Art of Arts, and Science of Sciences; or Divinity is the Mistresse upon which all Arts and Sciences are to attend as servants and handmaides. And why then not* Geometry? *But in what place she should follow her, I dare not say: For I am no herald, and therefore I meddle not with precedencie: But if I were, she should be none of the hindermost of her traine.*

The Oratour saith, and very truly doubtlesse, That, Omnes

artes, quæ ad humanitatē pertinent, habent commune quoddam vinculum, & cognatione quadam inter se continentur. *All Arts which pertaine unto humanity, they have a certaine common bond, and are knit together by a kinde of affinity. If then any Arts and Sciences may be thought necessary attendants upon this great Lady; Then surely* Geometry *amongst the rest must needes be one: For otherwise her traine will be but loose and shattered.*

Plato *saith* τὸν θεὸν ἀκεὶ γεωμετρεῖν, *That God doth alwayes worke by* Geometry, *that is, as the wiseman doth interprete it,* Sap. XI. 21. Omnia in mensura & numero & pondere disponere. *Dispose all things by measure, and number, and weight: Or, as the learned* Plutarch *speaketh; He adorneth and layeth out all the parts of the world according to rate, proportion, and similitude. Now who, I pray you, understandeth what these termes meane, but he which hath some meane skill in* Geometry? *Therefore none but such an one, may be able to declare and teach these things unto others.*

How many things are there in holy Scripture which may not well be understood without some meane skill in Geometry? *The Fabricke and bignesse of* Noah's *Arke: The Sciagraphy of the Temple set out by* Ezechiel, *Who may understand, but he that is skilfull in these Arts? I speake not of many and sundry words both in the New and Old Testaments, whose genuine and proper signification is merely Geometricall: And cannot well be conceived but of a Geometer.*

And here, that I may speake it without offence, I would have

it observed, how many men, much magnified for learning, not onely in their speeches, which alwayes are not premeditated, but even in their writings, exposed to the view and censure of all men, doe often paralogizein*, speake much, and little to the purpose. This they could not so easily and often doe, if they had beene but meanely practised in these kinde of studies. Wherefore that Epigramme which was used to be written over their Philosophy Schoole doores,* οὐδῆις ἀγεωμέτρητος εἴσιτω, *No man ignorant of* Geometry *come within these doores: Now written over our Divinitie Schooles. And if any man shall thinke this an hard sentence, let him heare what Saint* Augustine *saith in the same case,* Nemo ad divinarum humanarumq; rerum cognitionem accedat, nisi prius annumerandi artem addiscat: *Let no man come neither within the Divinity nor Philosophy Schooles, except he have first learned Arithmeticke. Now that the one of these Arts cannot be learned without the other;* Euclide *our great Master, who made but one of both, hath sufficiently demonstrated.*

If I should alledge the like practise of famous Divines, greatly admired for their great skill in this profession, as T. Peckham *Arch-Bishop of Canterbury,* Maurolycus *Bishop of* Messana *in* Sicilia, Cusanus *Cardinall of* Rome, *and many others, before indifferent judges, I am sure I should not be condemned. Who doth not greatly magnifie the grave* Seb. Munster, *the nimble* Ph. Melanchthon, *and the noble* Bernardino Baldo *Abbot of* Guastill, *and the painefull* Barth. Pitiscus *of* Grunberg, *for their knowledge and paines in these Arts and Sciences? And thus much shall at this*

time suffice, to have spoken unto the first Question: If any shall require further satisfaction, those I referre unto the forenamed Authors, whose authority peradventure may more prevaile with them, then my reasons may.

The next is concerning mine Author, and the worke in hand Geometry, *it must needs be confest we are beholden to* Euclides *Elements for: And he that would be rich in that profession, may have, if he be not covetous, his fill there, if he will labour hard, and take paines for it, it is true. But in what time thinke yau, may a man learne all* Euclide, *and so by him be made skilfull in this Art? By himselfe I know not whether ever or never: And with the helpe of another, although very expert, I will not promise him that hee shall attaine to perfection in many yeares.*

Hippocrates *the Prince of Physicians hath, as they say, in his workes laid out the whole Art of Physicke; but I marvell how long a man should study him alone, and read him over and over, before he should be a good Physician? I feare mee all the friends that he hath, and neighbours round about him, yea, and himselfe too, would all die before he should be able to hele them, or per adventure ere he should be able to know what they ail'd; and after 30, or 40. yeeres of such his study, I would be very loath to commit my selfe unto him. How much therefore are the students of this noble Science beholding unto those men, who by their industry, practise, and painefull travells, have shewed them a ready and certaine way through this wildernesse?*

The Elements of Euclide *they do containe generally the whole*

art of Geometry: *But if you will offer to travell thorow them alone, you shall finde them, I will warrant you, Elements indeed: for there you may walke through the spacious Aire, and over the great and wide sea, and in and about the vaste and arid wildernesse many a day and night, before you shall know where you are. This* Ramus, *my Authour in reading him found to be true; and confesseth himselfe often to have beene at a stand: Often to have lost himselfe: Often to have hitte upon a rocke, when he had thought he had touch'd land.*

Least therefore other men, in this journey doe not likewise loose themselves, for the benefit and safety, I meane, of others he hath prick'd them out a charde or chack'd out a way, which if thou shalt please to follow, it shall lead thee to thy wayes end, as directly, and in as short time, as conveniently may be. Yet in what time I cannot warrant thee: For all mens capacity, especially in these Arts, is not alike: All are not a like painefull, industrious, or diligent: All are not of the same ability of body, to be able to continue or sit at it: Or all not so free from other imployments or businesse calling them from their study, as some others are. For know this for certaine, Thou shalt here make no great progresse, except thou doe make it as it were a continued labour, Here you must observe that rule of the great Painter, Nulla dies sine linea, *Let no day passe over your head, in which you draw not some diagram or figure or other.*

One other thing let me also advise thee of, how capable soever thou art, refuse not, if thou maist have it, the helpe of a teacher; For except thou be another Hippocrates *or* Forcatelus, *whō our*

Authour mentioneth, thou canst not in these Arts and Sciences attaine unto any great perfection without infinite patience and great losse of most precious time, For they are therefore called Μαθηματικόι, *Mathematicks, that is, doctrinal or disciplinary Arts, because they are not to be attained unto by our owne information and industry; but by the helpe and instruction of others.*

This Worke gentle Reader, was in part above 30. yeares since published by M. Thomas Hood, *a learned man, and loving friend of mine, who teaching these Arts, in the Staplers Chappell in Leadenhall London, for the benefit of his Schollers and Auditory, did set out the Elements apart by themselves. The whole at large, with the Diagrammes, and Demonstrations, hee promised, as appeareth in the Preface to that his Worke, at his convenient leysure to send out shortly, after them. This for ought we know or can learne, is not by him or any other performed: And yet are those alone, without these of small use or none to a learner, where a teacher is not alwayes at hand. Wherefore we are bold being (encouraged thereunto by some private friends, and especially by the learned M.* H. Brigges, *professour of* Geometry *in the famous Vniversity of* Oxford) *to publish this of ours long since finished and ended.*

The usuall termes, whether Latine or Greeke, commonly used by the Geometers, *we have set downe and expressed in English, as well as we could, as others, writing of this argument in our language, have done before us. These termes, I doubt not, may*

by some in English otherwise be expressed, but how harsh those termes, may unto Mathematicall eares, at the first appeare, I will not say; and use in short time will make these familiar, and as pleasing to the eare as those possibly may be.

Our Authour, in the declaration of the Elements hath many passages, which in our judgement doe not make so much for the understanding of the matter in hand, as for the defence of the method here used, against Aristotle, Euclide, Proclus, *and others, which we have therfore wholly omitted. Some other things, which in our opinion, might in some respect illustrate any particular in this businesse, we have here and there inserted. Out of the learned* Finkius's Geometria Rotundi, *Wee have added to the fifth Booke certaine Propositions with their Consectaries out of* Ptolomi's *Almagest. The painfull and diligent* Rod. Snellius *out of the Lectures and Annotations of B.* Salignacus, I. Tho. Freigius, *and others, hath illustrated and altered here and there some few things.*

THE FIRST BOOKE OF *Peter Ramus's* Geometry, *Which is of a Magnitude*

1. *Geometry is the Art of measuring well.*

The end or scope of Geometry is to measure well: Therefore it is defined of the end, as generally all other Arts are. *To measure well* therefore is to consider the nature and affections of every thing that is to be measured: To compare such like things one with another: And to understand their reason and proportion and similitude. For all that is to measure well, whether it bee that by Congruency and application of some assigned measure: Or by Multiplication of the termes or bounds: Or by Division of the product made by multiplication: Or by any other way whatsoever the affection of the thing to be measured be considered.

But this end of Geometry will appeare much more beautifull and glorious in the use and geometricall workes and practise then by precepts, when thou shalt observe Astronomers, Geographers, Land-meaters, Sea-men, Enginers, Architects, Carpenters, Painters, and Carvers, in the description and measuring of the Starres, Countries, Lands, Engins, Seas, Buildings, Pictures, and Statues or Images to use the helpe of no other art but of Geometry. Wherefore here the name of this art commeth farre short of the thing meant by it. (For *Geometria*,

made of *Gè*, which in the Greeke language signifieth the Earth; and *Métron*, a measure, importeth no more, but as one would say *Land-measuring*. And *Geometra*, is but *Agrimensor*, A land-meter: or as *Tully* calleth him *Decempedator*, a Pole-man: or as *Plautus*, *Finitor*, a Marke-man.) when as this Art teacheth not only how to measure the Land or the Earth, but the Water, and the Aire, yea and the whole World too, and in it all Bodies, Surfaces, Lines, and whatsoever else is to bee measured.

Now *a Measure*, as *Aristotle* doth determine it, in every thing to be measured, is some small thing conceived and set out by the measurer; and of the Geometers it is called *Mensura famosa*, a knowne measure. Which kinde of measures, were at first, as *Vitruvius* and *Herodo* teache us, taken from mans body: whereupon *Protagoras* sayd, *That man was the measure of all things*, which speech of his, Saint *Iohn*, *Apoc.* 21. 17. doth seeme to approve. True it is, that beside those, there are some other sorts of measures, especially greater ones, taken from other things, yet all of them generally made and defined by those. And because the stature and bignesse of men is greater in some places, then it is ordinarily in others, therefore the measures taken from them are greater in some countries, then they are in others. Behold here a catalogue, and description of such as are commonly either used amongst us, or some times mentioned in our stories and other bookes translated into our English tongue.

Granum hordei, a Barley corne, like as a wheat corne in weights, is no kinde of measure, but is *quiddam minimum in*

mensura, some least thing in a measure, whereof it is, as it were, made, and whereby it is rectified.

Digitus, a Finger breadth, conteineth 2. barly cornes length, or foure layd side to side:

Pollex, a Thumbe breadth; called otherwise *Vncia*, an ynch, 3. barley cornes in length:

Palmus, or *Palmus minor*, an Handbreadth, 4. fingers, or 3. ynches.

Spithama, or *Palmus major*, a Span, 3. hands breadth, or 9. ynches.

Cubitus, a Cubit, halfe a yard, from the elbow to the top of the middle finger, 6. hands breadth, or two spannes.

Ulna, from the top of the shoulder or arme-hole, to the top of the middle finger. It is two folde; A yard and an Elne. *A yard*, containeth 2. cubites, or 3. foote: *An Elne*, one yard and a quarter, or 2. cubites and ½.

Pes, a Foot, 4. hands breadth, or twelve ynches.

Gradus, or *Passus minor*, a Steppe, two foote and an halfe.

Passus, or *Passus major*, a Stride, two steppes, or five foote.

Pertica, a Pertch, Pole, Rod or Lugge, 5. yardes and an halfe.

Stadium, a Furlong; after the Romans, 125. pases: the English, 40. rod.

Milliare, or *Milliarium*, that is *mille passus*, 1000. passes, or 8. furlongs.

Leuca, a League, 2. miles: used by the French, spaniards, and seamen.

Parasanga, about 4. miles: a Persian, & common Dutch mile; 30. furlongs.

Schœnos, 40. furlongs: an Egyptian, or swedland mile.

Now for a confirmation of that which hath beene saide, heare the words of the Statute.

It is ordained, That 3. graines of Barley, dry and round, do make an Ynch: 12. *ynches do make a* Foote: 3. *foote do make a* Yard: *5. yardes and ½ doe make a Perch: And 40. perches in length, and 4. in breadth, doe make an Aker: 33. Edwar. 1. De terris mensurandis: & De compositione ulnarum & Perticarum.*

Item, *Bee it enacted by the authority aforesaid; That a* Mile *shall be taken and reckoned in this manner, and no otherwise; That is to say,* a Mile *to containe 8. furlongs: And every* Furlong *to containe 40. lugges or poles: And every* Lugge *or* Pole *to containe 16. foote and ½. 25. Eliza.* An Act for restraint of new building, &c.

These, as I said, are according to diverse countries, where they are used, much different one from another: which difference, in my judgment; ariseth especially out of the difference of the Foote, by which generally they are all made, whether they be greater of lesser. For the Hand being as before hath beene taught, the fourth part of the foot whether greater or lesser: And the Ynch, the third part of the hand, whether greater or lesser.

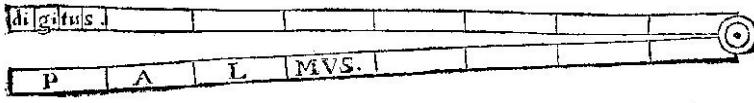
Item, the Yard, containing 3. foote, whether greater or lesser: And the Rodde 5. yardes and ½, whether greater or lesser, and so forth of the rest; It must needes follow, that the Foote

beeing in some places greater then it is in other some, these measures, the Hand, I meane, the Ynch, the Yard, the Rod, must needes be greater or lesser in some places then they are in other. Of this diversity therefore, and difference of the foot, in forreine countries, as farre as mine intelligence will informe me, because the place doth invite me, I will here adde these few lines following. For of the rest, because they are of more speciall use, I will God willing, as just occasion shall be administred, speake more plentifully hereafter.

Of this argument divers men have written somewhat, more or lesse: But none to my knowledge, more copiously and curiously, then *Iames Capell*, a Frenchman, and the learned *Willebrand*, *Snellius*, of *Leiden* in Holland, for they have compared, and that very diligently, many and sundry kinds of these measures one with another. The first as you may see in his treatise *De mensuris intervallorum* describeth these eleven following: of which the greatest is *Pes Babylonius*, the Babylonian foote; the least, *Pes Toletanus*, the foote used about *Toledo* in Spaine: And the meane betweene both, *Pes Atticus*, that used about *Athens* in Greece. For they are one unto another as 20. 15. and 12. are one unto another. Therefore if the Spanish foote, being the least, be devided into 12. ynches, and every inch againe into 10. partes, and so the whole foote into 120. the *Atticke* foote shall containe of those parts 150. and the *Babylonian*, 200. To this *Atticke* foote, of all other, doth ours come the neerest: For our *English* foote comprehendeth almost 152. such parts.

The other, to witt the learned *Snellius*, in his *Eratosthenes Batavus*, a booke which hee hath written of the true quantity of the compasse of the Earth, describeth many more, and that after a farre more exact and curious manner.

Here observe, that besides those by us here set downe, there are certaine others by him mentioned, which as hee writeth are found wholly to agree with some one or other of these. For *Rheinlandicus*, that of *Rheinland* or *Leiden*, which hee maketh his base, is all one with *Romanus*, the *Italian* or *Roman* foote. *Lovaniensis*, that of *Lovane*, with that of *Antwerpe*: *Bremensis*, that of *Breme* in *Germany*, with that of *Hafnia*, in *Denmarke*. Onely his *Pes Arabicus*, the *Arabian* foote, or that mentioned in *Abulfada*, and *Nubiensis*: the Geographers I have overpassed, because hee dareth not, for certeine, affirme what it was.



Looke of what parts *Pes Tolitanus*, the spanish foote, or that of *Toledo* in Spaine, conteineth 120. of such is the *Pes*.

Heidelbergicus, that of Heidelberg, 137.
Hetruscus, that of Tuscan, in Italie, 138.
Sedanensis, of Sedan in France, 139.
Romanus, that of Rome in Italy, 144.
Atticus, of Athens in Greece, 150.

Anglicus, of England, 152.
Parisinus, of Paris in France, 160.
Syriacus, of Syria, 166.
Ægyptiacus, of Egypt, 171.
Hebraicus, that of Iudæa, 180.
Babylonius, that of Babylon, 200.

Looke of what parts *Pes Romanus*, the foote of Rome, (which is all one with the foote of *Rheinland*) is 1000. of such parts is the foote of

Toledo, in Spaine, 864.
Mechlin, in Brabant, 890.
Strausburgh, in Germany, 891.
Amsterdam, in Holland, 904.
Antwerpe, in Brabant, 909.
Bavaria, in Germany, 924.
Coppen-haun, in Denmarke, 934.
Goes, in Zeland, 954.
Middleburge, in Zeland, 960.
London, in England, 968.
Noremberge, in Germany, 974.
Ziriczee, in Zeland, 980.
The ancient *Greeke*, 1042.
Dort, in Holland, 1050.
Paris, in France, 1055.
Briel, in Holland, 1060.
Venice, in Italy, 1101.
Babylon, in Chaldæa, 1172.
Alexandria, in Egypt, 1200.

Antioch, in Syria, 1360.

Of all other therefore our English foote commeth neerest unto that used by the Greekes: And the learned Master *Ro. Hues*, was not much amisse, who in his booke or Treatise *De Globis*, thus writeth of it *Pedem nostrum Angli cum Græcorum pedi æqualem invenimus, comparatione facta cum Græcorum pede, quem Agricola & alij ex antiquis monumentis tradiderunt.*

Now by any one of these knowne and compared with ours, to all English men well knowne the rest may easily be proportioned out.

2. *The thing proposed to bee measured is a Magnitude.*

Magnitudo, a Magnitude or Bignesse is the subject about which Geometry is busied. For every Art hath a proper subject about which it doth employ al his rules and precepts: And by this especially they doe differ one from another. So the subject of Grammar was speech; of Logicke, reason; of Arithmeticke, numbers; and so now of Geometry it is a magnitude, all whose kindes, differences and affections, are hereafter to be declared.

3. *A Magnitude is a continuall quantity.*

A Magnitude is *quantitas continua*, a continued, or continuall quantity. A number is *quantitas discreta*, a disjoined quantity: As one, two, three, foure; doe consist of one, two, three, foure unities, which are disjoyned and severed parts: whereas the parts of a Line, Surface, and Body are contained and continued without any manner of disjunction, separation, or distinction at all, as by and by shall better and more plainely appeare. Therefore

a Magnitude is here understood to be that whereby every thing to be measured is said to bee great: As a Line from hence is said to be long, a Surface broade, a Body solid: Wherefore Length, Breadth, and solidity are Magnitudes.

4. *That is* continuum, *continuall, whose parts are contained or held together by some common bound*.

This definition of it selfe is somewhat obscure, and to be understand onely in a geometricall sense: And it dependeth especially of the common bounde. For the parts (which here are so called) are nothing in the whole, but in a *potentia* or powre: Neither indeede may the whole magnitude bee conceived, but as it is compact of his parts, which notwithstanding wee may in all places assume or take as conteined and continued with a common bound, which Aristotle nameth a *Common limit*; but *Euclide* a *Common section*, as in a line, is a Point, in a surface, a Line: in a body, a Surface.

5. *A bound is the outmost of a Magnitude*.

Terminus, a Terme, or Bound is here understood to bee that which doth either bound, limite, or end *actu*, in deede; as in the beginning and end of a magnitude: Or *potentia*, in powre or ability, as when it is the common bound of the continuall magnitude. Neither is the Bound a parte of the bounded magnitude: For the thing bounding is one thing, and the thing bounded is another: For the Bound is one distance, dimension, or degree, inferiour to the thing bounded: A Point is the bound of a line, and it is lesse then a line by one degree,

because it cannot bee divided, which a line may. A Line is the bound of a surface, and it is also lesse then a surface by one distance or dimension, because it is only length, wheras a surface hath both length and breadth. A Surface is the bound of a body, and it is lesse likewise then it is by one dimension, because it is onely length and breadth, whereas as a body hath both length, breadth, and thickenesse.

Now every Magnitude *actu*, in deede, is terminate, bounded and finite, yet the geometer doth desire some time to have an infinite line granted him, but no otherwise infinite or farther to bee drawane out then may serve his turne.

6. *A Magnitude is both infinitely made, and continued, and cut or divided by those things wherewith it is bounded.*

A line, a surface, and a body are made gemetrically by the motion of a point, line, and surface: Item, they are conteined, continued, and cut or divided by a point, line, and surface. But a Line is bounded by a point: a surface, by a line: And a Body by a surface, as afterward by their severall kindes shall be understood.

Now that all magnitudes are cut or divided by the same wherewith they are bounded, is conceived out of the definition of *Continuum*, e. 4. For if the common band to containe and couple together the parts of a Line, surface, & Body, be a Point, Line, and Surface, it must needes bee that a section or division shall be made by those common bandes: And that to bee dissolved which they did containe and knitt together.

7. *A point is an undivisible signe in a magnitude.*

A Point, as here it is defined, is not naturall and to bee perceived by sense; Because sense onely perceiveth that which is a body; And if there be any thing lesse then other to be perceived by sense, that is called a Point. Wherefore a Point is no Magnitude: But it is onely that which in a Magnitude is conceived and imagined to bee undivisible. And although it be voide of all bignesse or Magnitude, yet is it the beginning of all magnitudes, the beginning I meane *potentiâ*, in powre.

8. *Magnitudes commensurable, are those which one and the same measure doth measure: Contrariwise, Magnitudes incommensurable are those, which the same measure cannot measure.* 1, 2. d. X.

Magnitudes compared betweene themselves in respect of numbers have Symmetry or commensurability, and Reason or rationality: Of themselves, Congruity and Adscription. But the measure of a magnitude is onely by supposition, and at the discretion of the Geometer, to take as pleaseth him, whether an ynch, an hand breadth, foote, or any other thing whatsoever, for a measure. Therefore two magnitudes, the one a foote long, the other two foote long, are commensurable; because the magnitude of one foote doth measure them both, the first once, the second twice. But some magnitudes there are which have no common measure, as the Diagony of a quadrate and his side, 116. p. X. *actu*, in deede, are *Asymmetra*, incommensurable: And yet they are *potentiâ*, by power, *symmetra*, commensurable, to witt by their quadrates: For the quadrate of the diagony is double to the

quadrate of the side.

9. *Rationall Magnitudes are those whose reason may bee expressed by a number of the measure given. Contrariwise they are irrationalls.* 5. d. X.

Ratio, Reason, Rate, or Rationality, what it is our Authour (and likewise *Salignacus*) have taught us in the first Chapter of the second booke of their Arithmetickes: Thither therefore I referre thee.

Data mensura, a Measure given or assigned, is of *Euclide* called *Rhetè*, that is spoken, (or which may be uttered) definite, certaine, to witt which may bee expressed by some number, which is no other then that, which as we said, was called *mensura famosa*, a knowne or famous measure.

Therefore Irrationall magnitudes, on the contrary, are understood to be such whose reason or rate may not bee expressed by a number or a measure assigned: As the side of the side of a quadrate of 20. foote unto a magnitude of two foote; of which kinde of magnitudes, thirteene sorts are mentioned in the tenth booke of *Euclides Elements*: such are the segments of a right line proportionally cutte, unto the whole line. The Diameter in a circle is rationall: But it is irrationall unto the side of an inscribed quinquangle: The Diagony of an Icosahedron and Dodecahedron is irrationall unto the side.

10. *Congruall or agreeable magnitudes are those, whose parts beeing applyed or laid one upon another doe fill an equall place.*

Symmetria, Symmetry or Commensurability and Rate were

from numbers: The next affections of Magnitudes are altogether geometricall.

Congruentia, Congruency, Agreeablenesse is of two magnitudes, when the first parts of the one doe agree to the first parts of the other, the meane to the meane, the extreames or ends to the ends, and lastly the parts of the one, in all respects to the parts, of the other: so Lines are congruall or agreeable, when the bounding, points of the one, applyed to the bounding points of the other, and the whole lengths to the whole lengthes, doe occupie or fill the same place. So Surfaces doe agree, when the bounding lines, with the bounding lines: And the plots bounded, with the plots bounded doe occupie the same place. Now bodies if they do agree, they do seeme only to agree by their surfaces. And by this kind of congruency do we measure the bodies of all both liquid and dry things, to witt, by filling an equall place. Thus also doe the moniers judge the monies and coines to be equall, by the equall weight of the plates in filling up of an equall place. But here note, that there is nothing that is onely a line, or a surface onely, that is naturall and sensible to the touch, but whatsoever is naturall, and thus to be discerned is corporeall.

Therefore

11. *Congruall or agreeable Magnitudes are equall.* 8. *ax. j.*

A lesser right line may agree to a part of a greater, but to so much of it, it is equall, with how much it doth agree: Neither is that axiome reciprocall or to be converted: For neither in deede are Congruity and Equality reciprocall or convertible. For

a Triangle may bee equall to a Parallelogramme, yet it cannot in all points agree to it: And so to a Circle there is sometimes sought an equall quadrate, although incongruall or not agreeing with it: Because those things which are of the like kinde doe onely agree.

12. *Magnitudes are described betweene themselves, one with another, when the bounds of the one are bounded within the boundes of the other: That which is within, is called the inscript: and that which is without, the Circumscript.*

Now followeth Adscription, whose kindes are Inscription and Circumscription; That is when one figure is written or made within another: This when it is written or made about another figure.

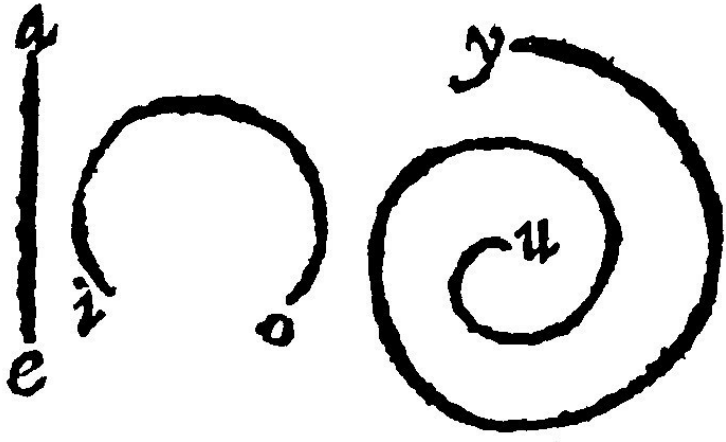
Homogenea, Homogenealls or figures of the same kinde onely betweene themselves *rectitermina*, or right bounded, are properly adscribed betweene themselves, and with a round. Notwithstanding, at the 15. booke of *Euclides Elements* Heterogenea, Heterogenealls or figures of divers kindes are also adscribed, to witt the five ordinate plaine bodies betweene themselves: And a right line is inscribed within a periphery and a triangle.

But the use of adscription of a rectilineall and circle, shall hereafter manifest singular and notable mysteries by the reason and meanes of adscripts; which adscription shall be the key whereby a way is opened unto that most excellent doctrine taught by the subtenses or inscripts of a circle as *Ptolomey* speakes, or Sines, as the latter writers call them.

The second Booke of *Geometry. Of a Line*

1. *A Magnitude is either a Line or a Lineate.*

The Common affections of a magnitude are hitherto declared: The *Species* or kindes doe follow: for other then this division our authour could not then meete withall.



2. *A Line is a Magnitude onely long.*

As are *ae. io.* and *uy.* such a like Magnitude is conceived in the measuring of waies, or distance of one place from another: And by the difference of a lightsome place from a darke: *Euclide*

at the 2 *d j.* defineth a line to be a length void of breadth: And indeede length is the proper difference of a line, as breadth is of a face, and solidity of a body.

3. *The bound of a line is a point.*

Euclide at the 3. *d j.* saith that the extremities or ends of a line are points. Now seeing that a Periphery or an hoope line hath neither beginning nor ending, it seemeth not to bee bounded with points: But when it is described or made it beginneth at a point, and it endeth at a pointe. Wherefore a Point is the bound of a line, sometime *actu*, in deed, as in a right line: sometime *potentiâ*, in a possibility, as in a perfect periphery. Yea in very deede, as before was taught in the definition of *continuum*, 4 *e.* all lines, whether they bee right lines, or crooked, are contained or continued with points. But a line is made by the motion of a point. For every magnitude generally is made by a geometricall motion, as was even now taught, and it shall afterward by the severall kindes appeare, how by one motion whole figures are made: How by a conversion, a Circle, Spheare, Cone, and Cylinder: How by multiplication of the base and heighth, rightangled parallelogrammes are made.

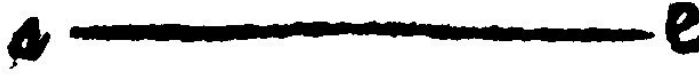
4. *A Line is either Right or Crooked.*

This division is taken out of the 4 d j. of *Euclide*, where rectitude or straightnes is attributed to a line, as if from it both surfaces and bodies were to have it. And even so the rectitude of a solid figure, here-after shall be understood by a right line perpendicular from the toppe unto the center of the base.

Wherefore rectitude is propper unto a line: And therefore also obliquity or crookednesse, from whence a surface is judged to be right or oblique, and a body right or oblique.

5. *A right line is that which lyeth equally betweene his owne bounds: A crooked line lieth contrariwise.* 4. *d. j.*

Now a line lyeth equally betweene his owne bounds, when it is not here lower, nor there higher: But is equall to the space comprehended betweene the two bounds or ends: As here *ae.* is, so hee that maketh *rectum iter*, a journey in a straight line, commonly he is said to treade so much ground, as he needes must, and no more: He goeth *obliquum iter*, a crooked way, which goeth more then he needeth, as *Proclus* saith.

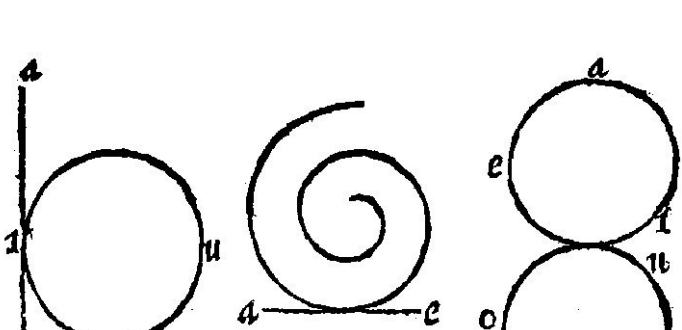


6. *A right line is the shortest betweene the same bounds.*

Linea recta, a straight or right line is that, as *Plato* defineth it, whose middle points do hinder us from seeing both the extremes at once; As in the eclipse of the Sunne, if a right line should be drawne from the Sunne, by the Moone, unto our eye, the body of the Moone beeing in the midst, would hinder our sight, and would take away the sight of the Sunne from us: which is taken from the Opticks, in which we are taught, that we see by straight

beames or rayes. Therfore to lye equally betweene the boundes, that is by an equall distance: to bee the shortest betweene the same bounds; And that the middest doth hinder the sight of the extremes, is all one.

7. *A crooked line is touch'd of a right or crooked line, when they both doe so meete, that being continued or drawne out farther they doe not cut one another.*



Tactus, Touching is propper to a crooked line, compared either with a right line or crooked, as is manifest out of the 2. and 3. *d* 3. A right line is said to touch a circle, which touching the circle and drawne out farther, doth not cut the circle, 2 *d* 3. as here *ae*, the right line toucheth the periphery *iou*. And *ae*. doth touch the helix or spirall. Circles are said to touch one another, when touching they doe not cutte one another, 3. *d* 3. as here the

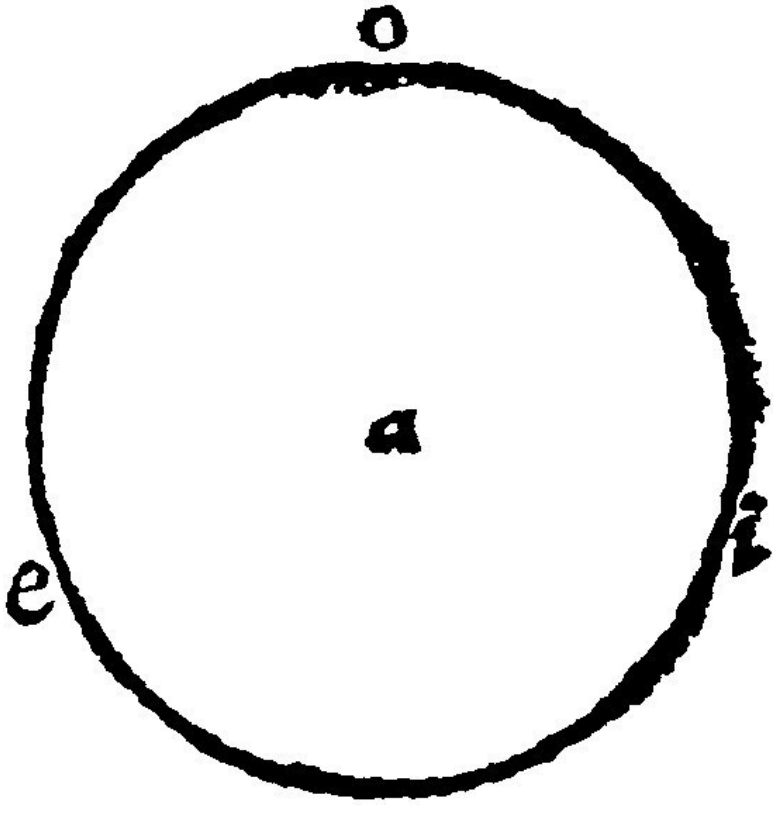
periphery doth *aej.* doth touch the periphery *ouy*.

Therefore

8. *Touching is but in one point onely. è 13. p* 3.

This Consectary is immediatly conceived out of the definition; for otherwise it were a cutting, not touching. So *Aristotle* in his *Mechanickes* saith; That a round is easiliest mou'd and most swift; Because it is least touch't of the plaine underneath it.

9. *A crooked line is either a Periphery or an Helix.* This also is such a division, as our Authour could then hitte on.

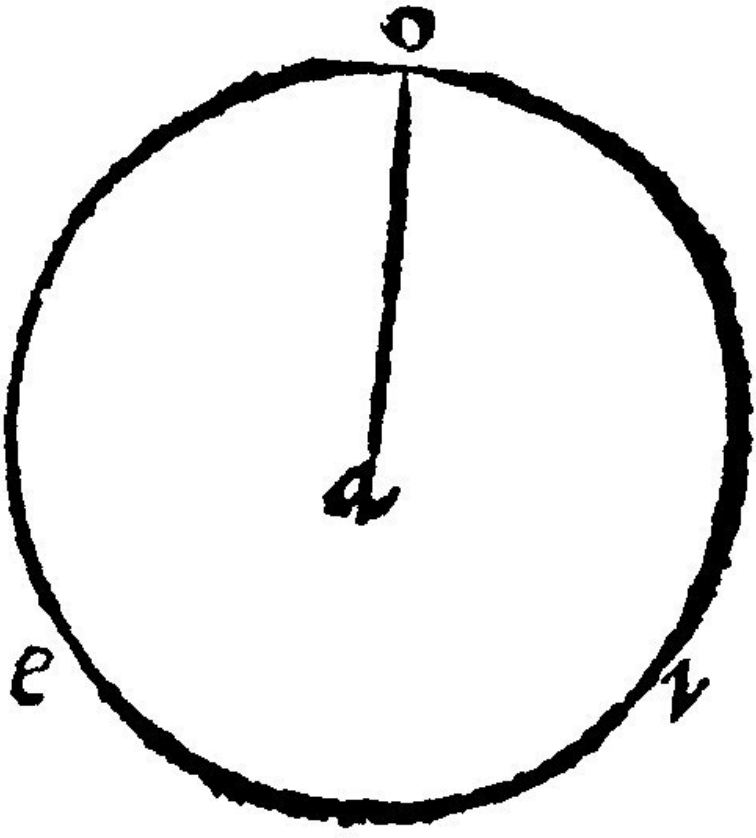


10. *A Periphery is a crooked line, which is equally distant from the middest of the space comprehended.*

Peripheria, a Periphery, or Circumference, as *eio.* doth stand equally distant from *a*, the middest of the space enclosed or conteined within it.

Therefore

11. *A Periphery is made by the turning about of a line, the one end thereof standing still, and the other drawing the line.*



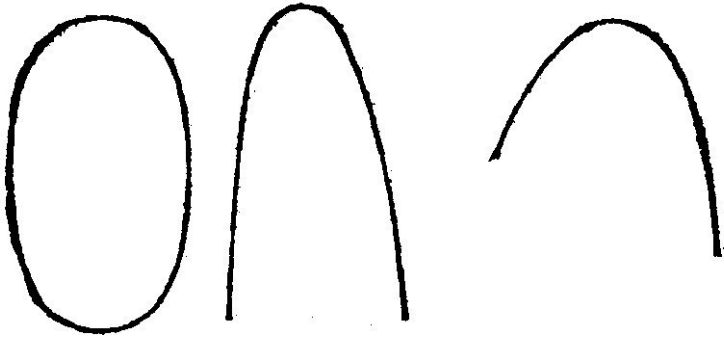
As in *eio.* let the point *a* stand still: And let the line *ao*, be turned about, so that the point *o* doe make a race, and it shall make the periphery *eoi*. Out of this fabricke doth *Euclide*, at the 15. d. j. frame the definition of a Periphery: And so doth hee afterwarde define a Cone, a Spheare, and a Cylinder.

Now the line that is turned about, may in a plaine, bee either a right line or a crooked line: In a sphericall it is onely a crooked line; But in a conicall or Cylindraceall it may bee a right line, as is the side of a Cone and Cylinder. Therefore in the conversion or turning about of a line making a periphery, there is considered onely the distance; yea two points, one in the center, the other in the toppe, which therefore Aristotle nameth *Rotundi principia*, the principles or beginnings of a round.

12. *An Helix is a crooked line which is unequally distant from the middest of the space, howsoever inclosed.*

Hæc tortuosa linea, This crankled line is of *Proclus* called *Helicoides*. But it may also be called *Helix*, a twist or wreath: The *Greekes* by this word do commonly either understand one of the kindes of Ivie which windeth it selfe about trees & other plants; or the strings of the vine, whereby it catcheth hold and twisteth it selfe about such things as are set for it to clime or run upon. Therfore it should properly signifie the spirall line. But as it is here taken it hath divers kindes; As is the *Arithmetica* which is Archimede'es Helix, as the *Conchois*, Cockleshell-like: as is the *Cittois*, Iuylike: The *Tetragonisousa*, the Circle squaring line, to witt that by whose meanes a circle may be brought into a square: The Admirable line, found out by *Menelaus*: The Conicall *Ellipsis*, the *Hyperbole*, the *Parabole*, such as these are, they

attribute to *Menechmus*: All these *Apollonius* hath comprised in eight Bookes; but being mingled lines, and so not easie to bee all reckoned up and expressed, *Euclide* hath wholly omitted them, saith *Proclus*, at the 9. *p. j.*

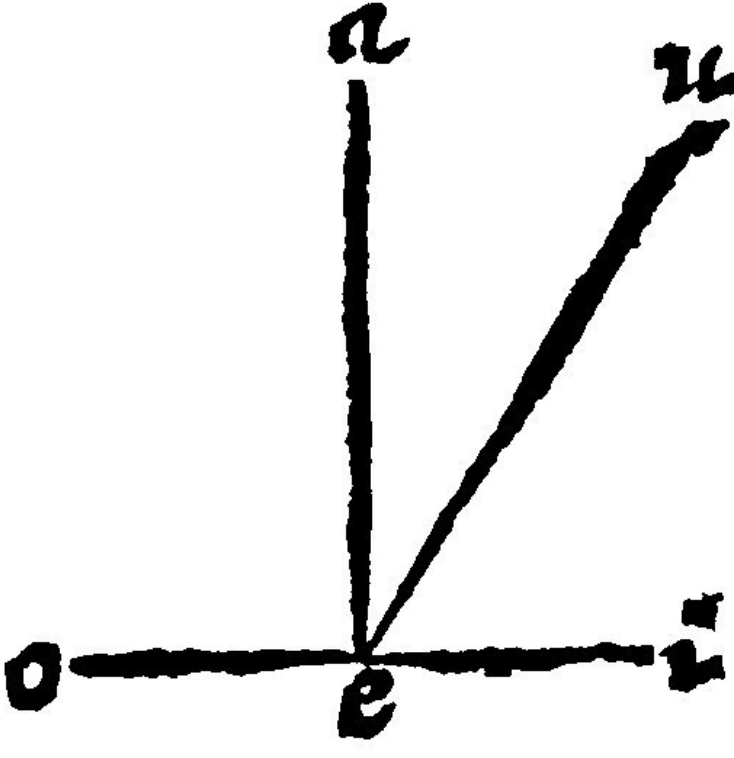


13. *Lines are right one unto another, whereof the one falling upon the other, lyeth equally: Contrariwise they are oblique. è 10. d j.*

Hitherto straightnesse and crookednesse have beene the affections of one sole line onely: The affections of two lines compared one with another are *Perpendiculum*, Perpendicularity and *Parallelismus*, Parallell equality; Which affections are common both to right and crooked lines. Perpendicularity is first generally defined thus:

Lines are right betweene themselves, that is, perpendicular one unto another, when the one of them lighting upon the other, standeth upright and inclineth or leaneth neither way. So two right lines in a plaine may bee perpendicular; as are *ae.* and *io.* so two peripheries upon a sphearicall may be perpendiculars, when the one of them falling upon the other, standeth indifferently betweene, and doth not incline or leane either way. So a right line may be perpendicular unto a periphery, if falling upon it, it doe reele neither way, but doe ly indifferently betweene either side. And in deede in all respects lines right betweene themselves, and perpendicular lines are one and the same. And from the perpendicularity of lines, the perpendicularity of surfaces is taken, as hereafter shall appeare. Of the perpendicularity of bodies, *Euclide* speaketh not one word in his *Elements*, & yet a body is judged to be right, that is, plumme or perpendicular unto another body, by a perpendicular line.

Therefore,



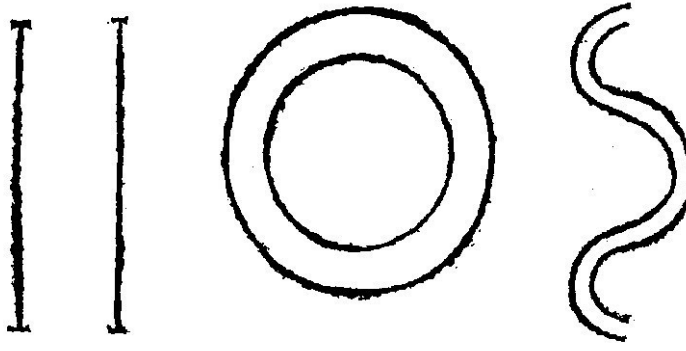
14. *If a right line be perpendicular unto a right line, it is from the same bound, and on the same side, one onely. ê 13. p. xj.*

Or, there can no more fall from the same point, and on the

same side but that one. This consectary followeth immediately upon the former: For if there should any more fall unto the same point and on the same side, one must needes reele, and would not ly indifferently betweene the parts cut: as here thou seest in the right line *ae. io. eu*.

15. *Parallell lines they are, which are everywhere equally distant. è 35. d j.*

Parallelismus, Parallell-equality doth now follow: And this also is common to crooked lines and right lines: As heere thou seest in these examples following.



Parallell-equality is derived from perpendicularity, and is of neere affinity to it. Therefore Posidonius did define it by a common perpendicle or plum-line: yea and in deed our definition intimateth asmuch. Parallell-equality of bodies is no where

mentioned in *Euclides Elements*: and yet they may also bee parallells, and are often used in the Optickes, Mechanickes, Painting and Architecture.

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Therefore,

16. *Lines which are parallell to one and the same line, are also parallell one to another*.

This element is specially propounded and spoken of right lines onely, and is demonstrated at the 30. *p. j.* But by an addition of equall distances, an equall distance is knowne, as here.

The third Booke of *Geometry*. Of an Angle

1. *A lineate is a Magnitude more then long.*

A New forme of doctrine hath forced our Authour to use oft times new words, especially in dividing, that the logicall lawes and rules of more perfect division by a dichotomy, that is into two kindes, might bee held and observed. Therefore a Magnitude was divided into two kindes, to witt into a Line and a Lineate: And a Lineate is made the *genus* of a surface and a Body. Hitherto a Line, which of all bignesses is the first and most simple, hath been described: Now followeth a Lineate, the other kinde of magnitude opposed as you see to a line, followeth next in order. *Lineatum* therefore a Lineate, or *Lineamentum*, a Lineament, (as by the authority of our Authour himselfe, the learned *Bernhard Salignacus*, who was his Scholler, hath corrected it) is that Magnitude in which there are lines: Or which is made of lines, or as our Authour here, which is more then long: Therefore lines may be drawne in a surface, which is the proper soile or plots of lines; They may also be drawne in a body, as the Diameter in a Prisma: the axis in a spheare; and generally all lines falling from aloft: And therfore *Proclus* maketh some plaine, other solid lines. So Conicall lines, as the Ellipsis, Hyperbole, and Parabole, are called solid lines because they do

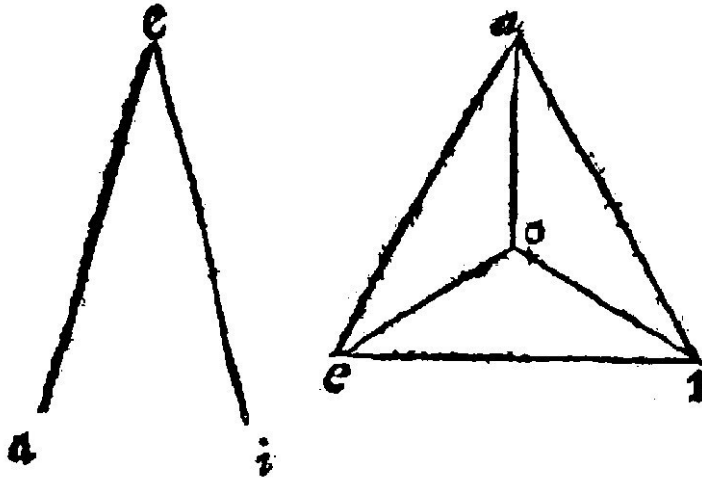
arise from the cutting of a body.

2. *To a Lineate belongeth an Angle and a Figure.*

The common affections of a Magnitude were to be bounded, cutt, jointly measured, and adscribed: Then of a line to be right, crooked, touch'd, turn'd about, and wreathed: All which are in a lineate by meanes of a line. Now the common affections of a Lineate are to bee Angled and Figured. And surely an Angle and a figure in all Geometricall businesses doe fill almost both sides of the leafe. And therefore both of them are diligently to be considered.

3. *An Angle is a lineate in the common section of the bounds.*

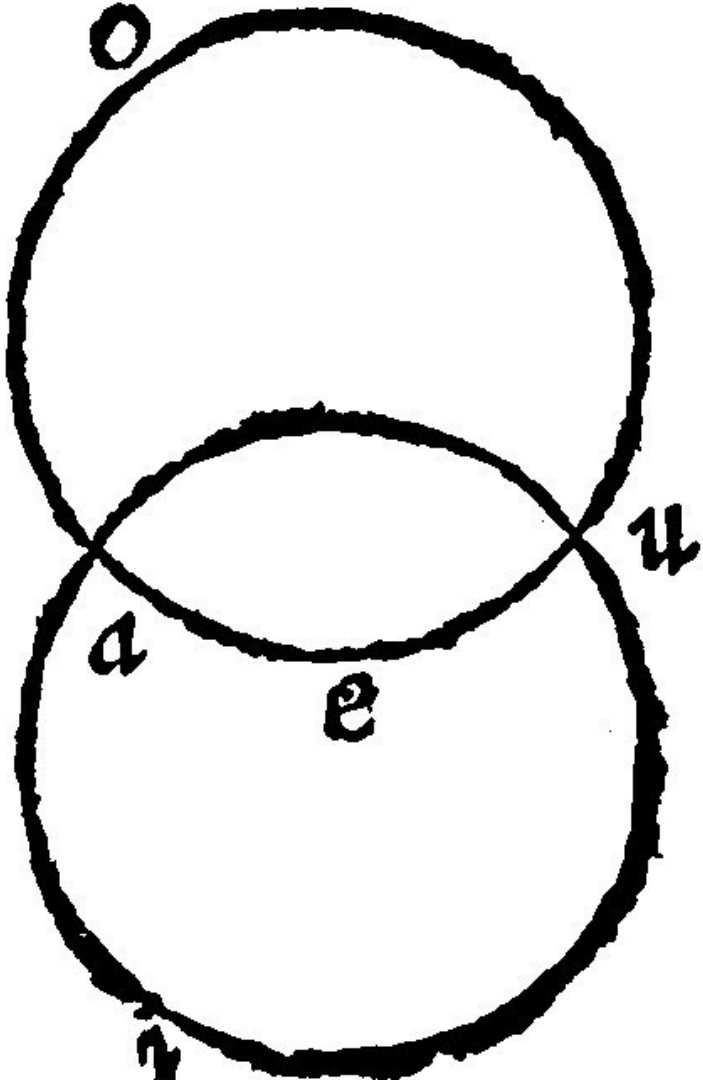
So *Angulus Superficiarius*, a superficiall Angle, is a surface consisting in the common section of two lines: So *angulus solidus*, a solid angle, in the common section of three surfaces at the least.



[But the learned B. *Salignacus* hath observed, that all angles doe not consist in the common section of the bounds, Because the touching of circles, either one another, or a rectilineal surface doth make an angle without any cutting of the bounds: And therefore he defineth it thus: *Angulus est terminorum inter se invicem inclinantium concursus: An angle is the meeting of bounds, one leaning towards another.*] So is *aei.* a superficiall angle: [And such also are the angles *ouy.* and *bcd.*] so is the angle *o.* a solid angle, to witt comprehended of the three surfaces *aoi. ioe.* and *aoe.* Neither may a surface, of 2. dimensions, be bounded with one right line: Nor a body, of three dimensions, bee bounded with two, at lest beeing plaine surfaces.

4. *The shankes of an angle are the bounds compreding the angle*.

Scèle or *Crura*, the Shankes, Legges, H. are the bounds insisting or standing upon the base of the angle, which in the Isosceles only or Equicrurall triangle are so named of *Euclide*, otherwise he nameth them *Latera*, sides. So in the examples aforesaid, *ea.* and *ei.* are the shankes of the superficiary angle *e*; And so are the three surfaces *aoi. ieo.* and *aeo.* the shankes of the said angle *o*. Therefore the shankes making the angle are either Lines or Surfaces: And the lineates formed or made into Angles, are either Surfaces or Bodies.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5. *Angles homogeneall, are angles of the same kinde, both in respect of their shankes, as also in the maner of meeting of the same:* [*Heterogeneall, are those which differ one from another in one, or both these conditions.*]

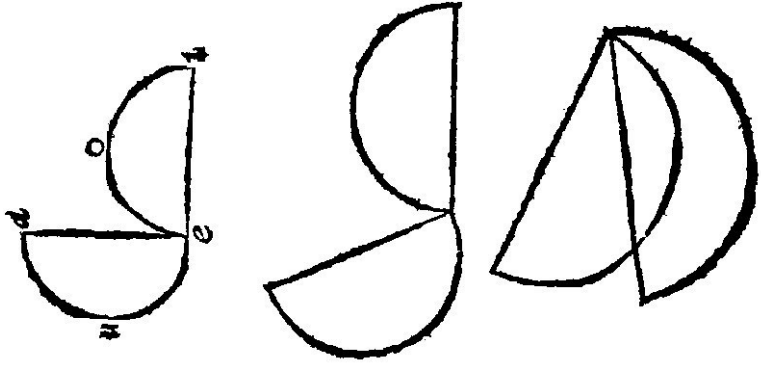
Therefore this *Homogenia*, or similitude of angles is twofolde, the first is of shanks; the other is of the manner of meeting of the shankes: so rectilineall right angles, are angles homogeneall betweene themselves. But right-lined right angles, and oblique-lined right angles between themselves, are heterogenealls. So are neither all obtusangles compared to all obtusangles: Nor all acutangles, to all acutangles, homogenealls, except both these conditions doe concurre, to witt the similitude both of shanke and manner of meeting. *Lunularis*, a Lunular, or Moonlike corner angle is homogeneall to a *Systroides* and *Pelecoides*, Hatchet formelike, in shankes: For each of these are comprehended of peripheries: The Lunular of one convexe; the other concave; as *iue*. The Systroides of both convex, as *iao*. The Pelecoides of both concave, as *eau*. And yet a lunular, in respect of the meeting of the shankes is both to the Systroides and Pelecoides heterogeneall: And therefore it is absolutely heterogeneall to it.

6. *Angels congruall in shankes are equall.*

This is drawne out of the 10. e j. For if twice two shanks doe agree, they are not foure, but two shankes, neither are they two equall angles, but one angle. And this is that which *Proclus* speaketh of, at the 4. p j. when hee saith, that a right lined angle

is equall to a right lined angle, when one of the shankes of the one put upon one of the shankes of the other, the other two doe agree: when that other shanke fall without, the angle of the out-falling shanke is the greater: when it falleth within, it is lesser: For there is comprehendeth; here it is comprehended.

Notwithstanding although congruall or agreeable angles be equall: yet are not congruity and equality reciprocall or convertible: For a Lunular may bee equall to a right lined right angle, as here thou seest: For the angles of equall semicircles *ieo.* and *aeu.* are equall, as application doth shew. The angle *aeo.* is common both to the right angle *aei.* and to the lunar *aueo.* Let therefore the equall angle *aeo.* bee added to both: the right angle *aei.* shall be equall to the Lunular *aueo*.



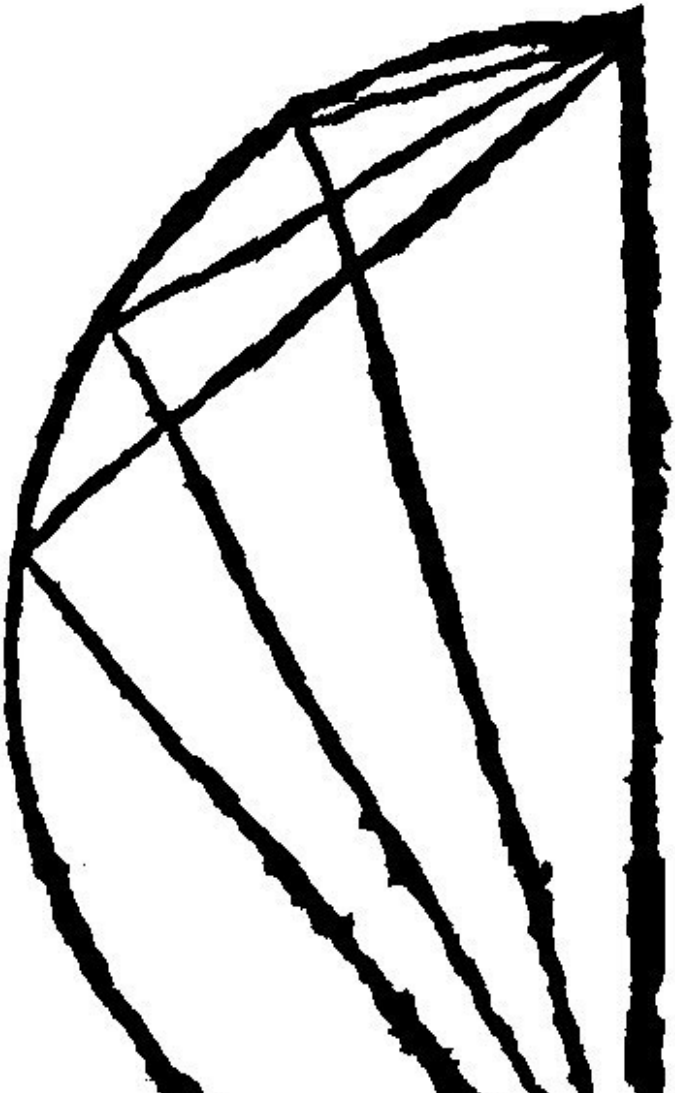
The same Lunular also may bee equall to an obtusangle and

Acutangle, as the same argument will demonstrate.

Therefore,

7. *If an angle being equicrurall to an other angle, be also equall to it in base, it is equall: And if an angle having equall shankes with another, bee equall to it in the angle, it is also equall to it in the base. è* 8. & 4. *p j.*

For such angles shall be congruall or agreeable in shanks, and also congruall in bases. *Angulus isosceles*, or *Angulus æquicrurus*, is a triangle having equall shankes unto another.



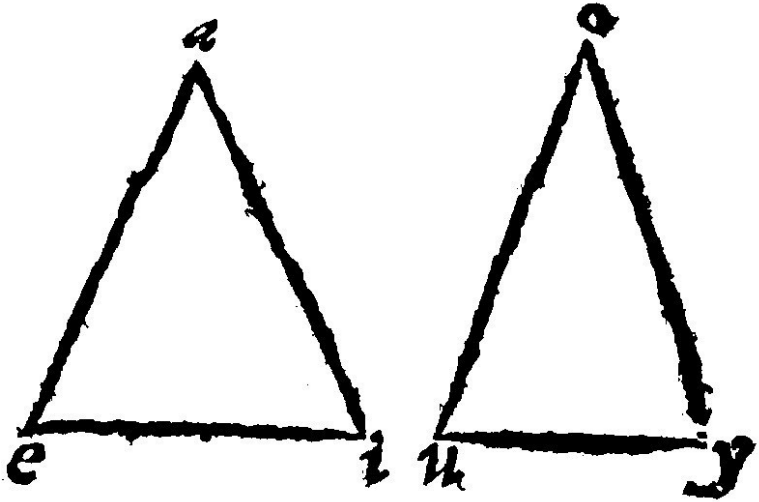
8. *And if an angle equall in base to another, be also equall to it in shankes, it is equall to it.*

For the congruency is the same: And yet if equall angles bee equall in base, they are not by and by equicrurall, as in the angles of the same section will appeare, as here. And so of two equalities, the first is reciprocall: The second is not. [And therefore is this Consectary, by the learned B. *Salignacus*, justly, according to the judgement of the worthy Rud. *Snellius*, here cancelled; or quite put out: For angles may be equall, although they bee unequall in shankes or in bases, as here, the angle *a.* is not greater then the angle *o*, although the angle *o* have both greater shankes and greater base then the angle *a*.]

And

9. *If an angle equicrurall to another angle, be greater then it in base, it is greater: And if it be greater, it is greater in base: è* 52 & 24. *p j.*

As here thou seest; [The angles *eai.* and *uoy.* are equicrurall, that is their shankes are equall one to another; But the base *ei* is greater then the base *uy*: Therefore the angle *eai*, is greater then the angle *uoy*. And contrary wise, they being equicrurall, and the angle *eai.* being greater then the angle *uoy.* The base *ei.* must needes be greater then the base *uy*.]



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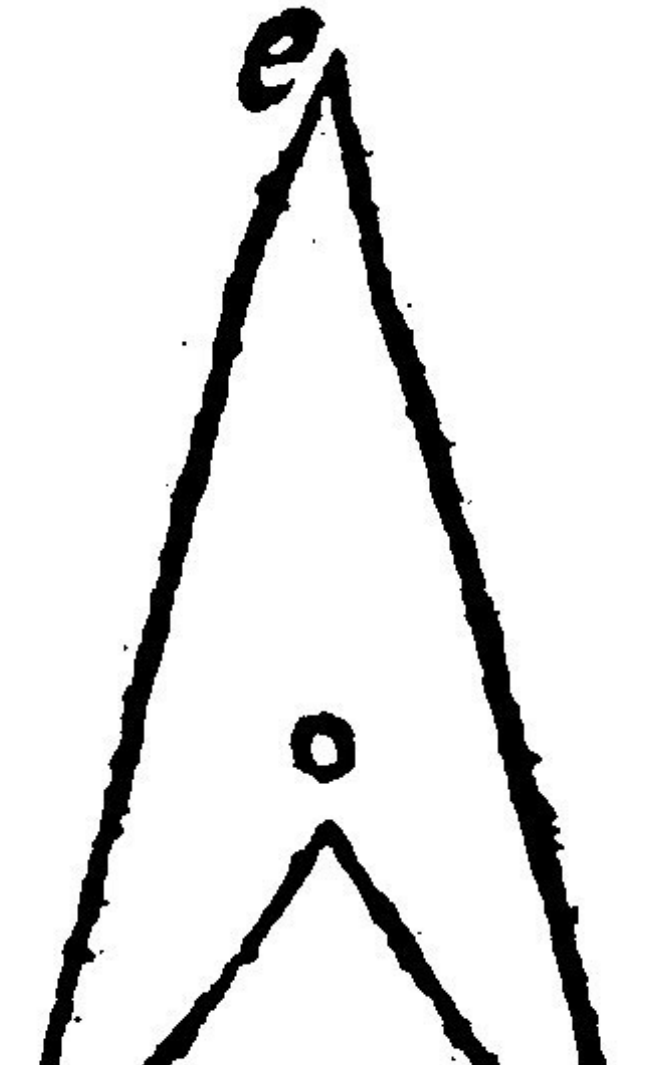
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10. *If an angle equall in base, be lesse in the inner shankes, it is greater.*

Or as the learned Master *T. Hood* doth paraphrastically translate it. *If being equall in the base, it bee lesser in the feete (the feete being conteined within the feete of the other angle) it is the greater angle.* [That is, if one angle enscribed within another angle, be equall in base, the angle of the inscribed shall be greater then the angle of the circumscribed.]

As here the angle *aoi.* within the angle *aei.* And the bases are equall, to witt one and the same; Therefore *aoi.* the inner angle is greater then *aei.* the outter angle. *Inner* is added of necessity: For otherwise there will, in the section or cutting one of another, appeare a manifest errour. All these consectaries are drawne out of that same axiome of congruity, to witt out of the 10. e j. as *Proclus* doth plainely affirme and teach: It seemeth saith hee, that the equalities of shankes and bases, doth cause the equality of the verticall angles. For neither, if the bases be equall, doth the equality of the shankes leave the same or equall angles: But if the base bee lesser, the angle decreaseth: If greater, it increaseth. Neither if the bases bee equall, and the shankes unequall, doth the angle remaine the same: But when they are made lesse, it is increased: when they are made greater, it is diminished: For the contrary falleth out to the angles and shankes of the angles. For if thou shalt imagine the shankes to be in the same base thrust downeward, thou makest them lesse, but their angle greater: but

if thou do againe conceive them to be pul'd up higher, thou makest them greater, but their angle lesser. For looke how much more neere they come one to another, so much farther off is the toppe removed from the base: wherefore you may boldly affirme, that the same base and equall shankes, doe define the equality of Angels. This *Poclus*,

Therefore,

11. *If unto the shankes of an angle given, homogeneall shankes, from a point assigned, bee made equall upon an equall base, they shall comprehend an angle equall to the angle given. è 23. p j.* & 26. *p xj.*

[This consectary teacheth how unto a point given, to make an angle equall to an Angle given. To the effecting and doing of each three things are required; First, that the shankes be homogeneall, that is in each place, either straight or crooked: Secondly, that the shankes bee made equall, that is of like or equall bignesse: Thirdly, that the bases be equall: which three conditions if they doe meete, it must needes be that both the angles shall bee equall: but if one of them be wanting, of necessity againe they must be unequall.]

This shall hereafter be declared and made plaine by many and sundry practises: and therefore here we bring no example of it.

12. *An angle is either right or oblique.*

Thus much of the Affections of an angle; the division into his kindes followeth. An angle is either Right or Oblique: as afore, at the 4 *e ij.* a line was right or straight, and oblique or crooked.

13. *A right angle is an angle whose shankes are right (that is perpendicular) one unto another: An Oblique angle is contrary to this.*

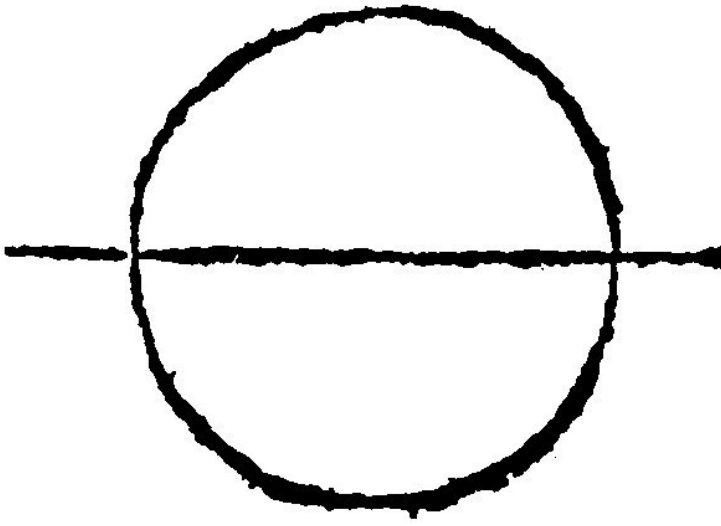
As here the angle *aio.* is a right angle, as is also *oie.* because the shanke *oi.* is right, that is, perpendicular to *ae.* [The instrument wherby they doe make triall which is a right angle, and which is oblique, that is greater or lesser then a right angle, is the square which carpenters and joyners do ordinarily use: For lengthes are tried, saith *Vitruvius*, by the Rular and Line: Heighths, by the Perpendicular or Plumbe: And Angles, by the square.] Contrariwise, an Oblique angle it is, when the one shanke standeth so upon another, that it inclineth, or leaneth more to one side, then it doth to the other: And one angle on the one side, is greater then that on the other.

Therefore,

14. *All straight-shanked right angles are equall.*

[That is, they are alike, and agreeable, or they doe fill the same place; as here are *aio.* and *eio.* And yet againe on the contrary: All straight shanked equall angles, are not right-angles.]

The axiomes of the equality of angles were three, as even now wee heard, one generall, and two Consectaries: Here moreover is there one speciall one of the equality of Right angles.



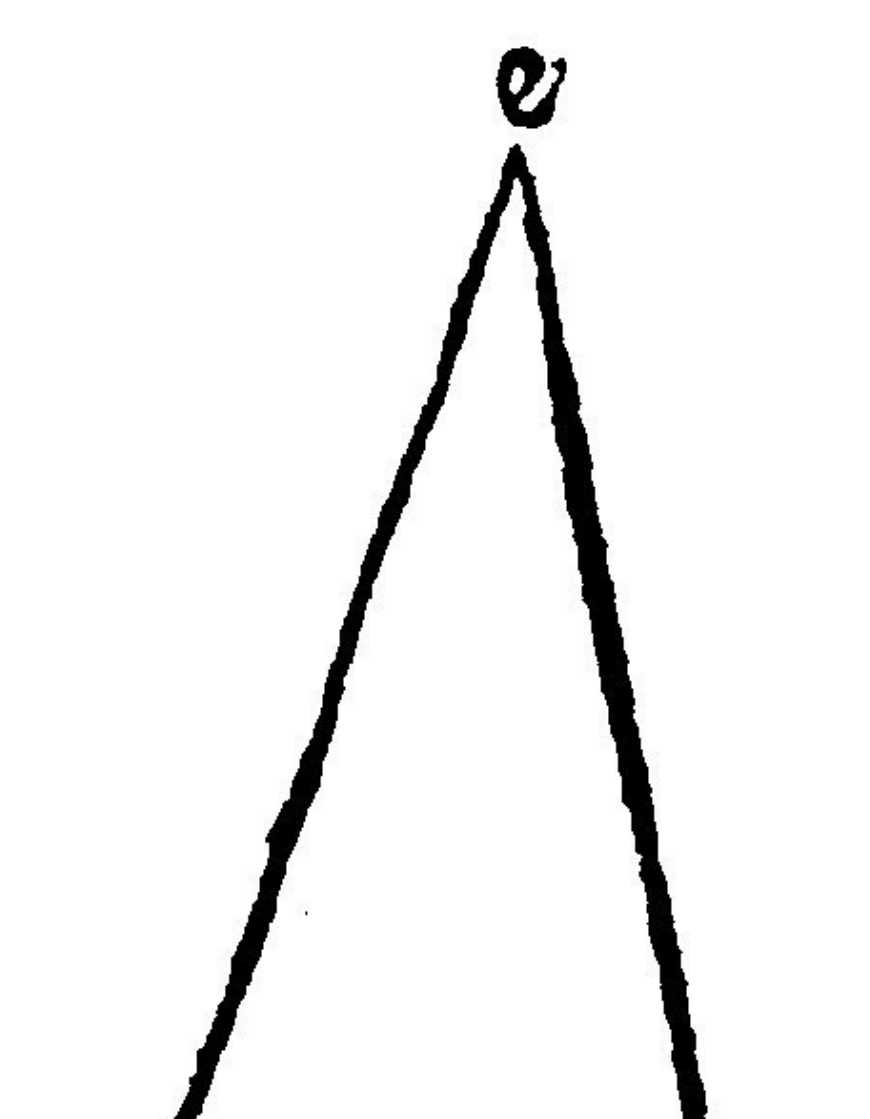
Angles therfore homogeneall and recticrurall, that is whose shankes are right, as are right lines, as plaine surfaces (For let us so take the word) are equall right angles. So are the above written rectilineall right angles equall: so are plaine solid right angles, as in a cube, equall. The axiome may therefore generally be spoken of solid angles, so they be recticruralls: Because all semicircular right angles are not equall to all semicircular right angles: As here, when the diameter is continued it is perpendicular, and maketh twice two angles, within and without, the outter equall betweene themselves, and inner equall betweene themselves: But the outer unequall to the inner: And the angle of

a greater semicircle is greater, then the angle of a lesser. Neither is this affection any way reciprocall, That all equall angles should bee right angles. For oblique angles may bee equall betweene themselves: And an oblique angle may bee made equall to a right angle, as a Lunular to a rectilineall right angle, as was manifest, at the 6 e.

The definition of an oblique is understood by the obliquity of the shankes: whereupon also it appeareth; That an oblique angle is unequall to an homogeneall right angle: Neither indeed may oblique angles be made equall by any lawe or rule: Because obliquity may infinitly bee both increased and diminished.

15. *An oblique angle is either Obtuse or Acute.*

One difference of Obliquity wee had before at the 9 e ij. in a line, to witt of a periphery and an helix; Here there is another dichotomy of it into obtuse and acute: which difference is proper to angles, from whence it is translated or conferred upon other things and metaphorically used, as *Ingenium obtusum, acutum*; A dull, and quicke witte, and such like.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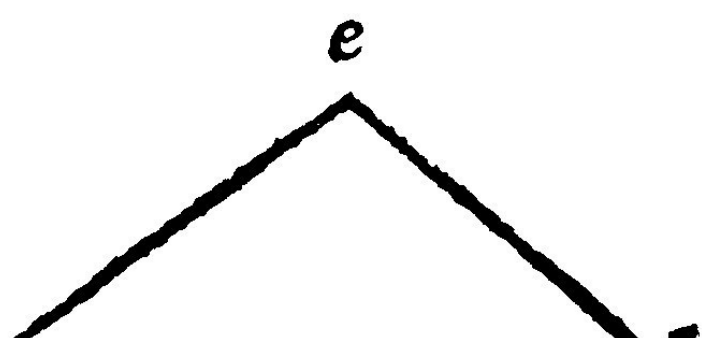


16. *An obtuse angle is an oblique angle greater then a right angle. 11. d j.*

Obtusus, Blunt or Dull; As here *aei.* In the definition the *genus* of both *Species* or kinds is to bee understood: For a right lined right angle is greater then a sphearicall right angle, and yet it is not an obtuse or blunt angle: And this greater inequality may infinitely be increased.

17. *An acutangle is an oblique angle lesser then a right angle. 12. d j.*

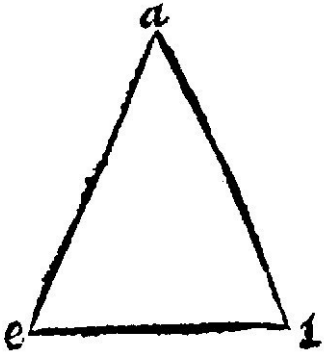
Acutus, Sharpe, Keene, as here *aei.* is. Here againe the same *genus* is to bee understood: because every angle which is lesse then any right angle is not an acute or sharp angle. For a

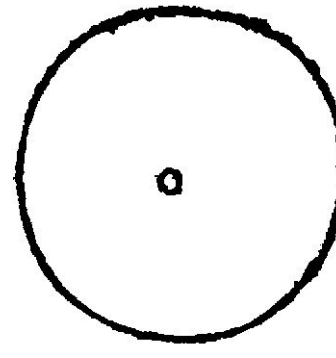
semicircle and sphericall right angle, is lesse then a rectilineall right angle, and yet it is not an acute angle.

The fourth Booke, which is of a Figure

1. *A figure is a lineate bounded on all parts.*

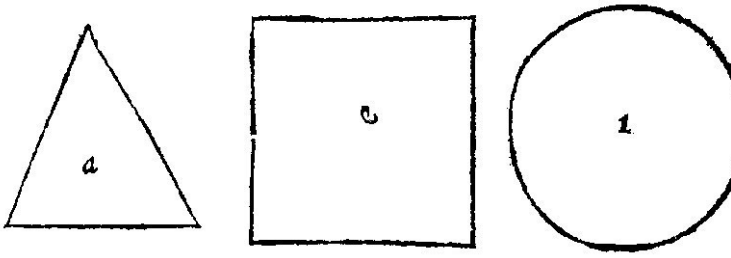
So the triangle *aei.* is a figure; Because it is a plaine bounded on all parts with three sides. So a circle is a figure: Because it is a plaine every way bounded with one periphery.





2. *The center is the middle point in a figure.*

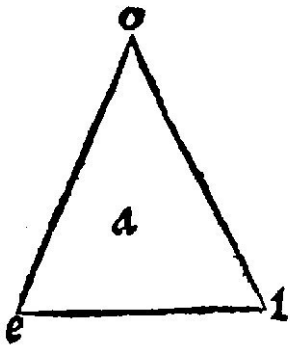
In some part of a figure the Center, Perimeter, Radius, Diameter and Altitude are to be considered. The Center therefore is a point in the midst of the figure; so in the triangle, quadrate, and circle, the center is, *aei.*

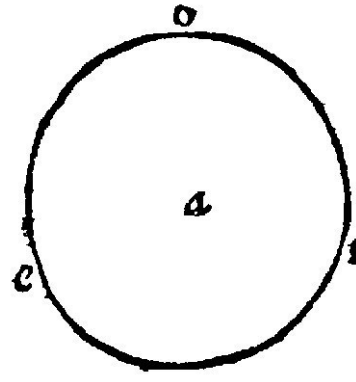


Centrum gravitatis, the center of weight, in every plaine magnitude is said to bee that, by the which it is handled or held up parallell to the horizon: Or it is that point whereby the weight being suspended doth rest, when it is caried. Therefore if any plate should in all places be alike heavie, the center of magnitude and weight would be one and the same.

3. *The perimeter is the compasse of the figure.*

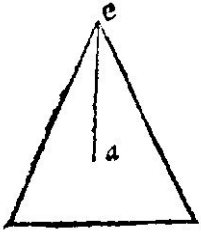
Or, the perimeter is that which incloseth the figure. This definition is nothing else but the interpretation of the Greeke word. Therefore the perimeter of a Triangle is one line made or compounded of three lines. So the perimeter of the triangle *a*, is *eio.* So the perimeter of the circle *a* is a periphery, as in *eio.* So the perimeter of a Cube is a surface, compounded of sixe surfaces: And the perimeter of a spheare is one whole sphæricall surface, as hereafter shall appeare.

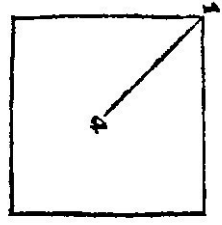


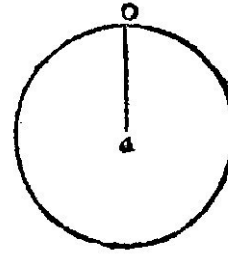


4. *The Radius is a right line drawne from the center to the perimeter.*

Radius, the Ray, Beame, or Spoake, as of the sunne, and cart wheele: As in the figures under written are *ae*, *ai*, *ao*. It is here taken for any distance from the center, whether they be equall or unequall.

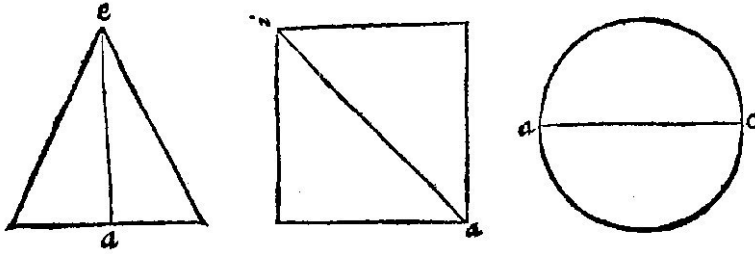






5. *The Diameter is a right line inscribed within the figure by his center.*

As in the figure underwritten are *ae*, *ai*, *ao*. It is called the *Diagonius*, when it passeth from corner to corner. In solids it is called the *Axis*, as hereafter we shall heare.



Therefore,

6. *The diameters in the same figure are infinite.*

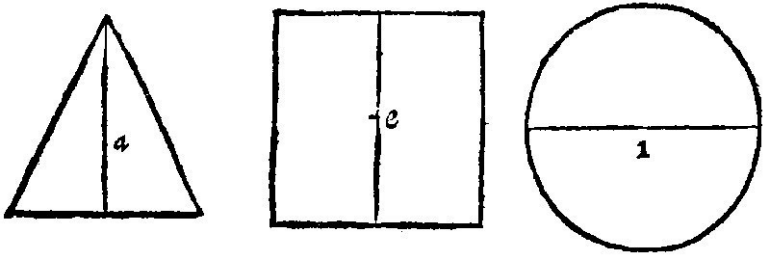
Although of an infinite number of unequall lines that be only the diameter, which passeth by or through the center notwithstanding by the center there may be divers and sundry. In a circle the thing is most apparent: as in the Astrolabe the index may be put up and downe by all the points of the periphery. So in a speare and all rounds the thing is more easie to be conceived, where the diameters are equall: yet notwithstanding in other figures the thing is the same. Because the diameter is a right line inscribed by the center, whether from corner to corner, or side

to side, the matter skilleth not. Therefore that there are in the same figure infinite diameters, it issueth out of the difinition of a diameter.

And

7. *The center of the figure is in the diameter.*

As here thou seest *a, e, i* this ariseth out of the definition of the diameter. For because the diameter is inscribed into the figure by the center: Therefore the Center of the figure must needes be in the diameter thereof: This is by *Archimedes* assumed especially at the 9, 10, 11, and 13 *Theoreme* of his *Isorropicks*, or *Æquiponderants*.



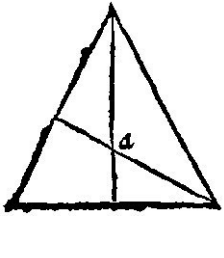
This consectary, saith the learned Rod. Snellius, is as it were a kinde of invention of the center. For where the diameters doe meete and cutt one another, there must the center needes bee. The cause of this is for that in every figure there is but one center only: And all the diameters, as before was said, must needes passe

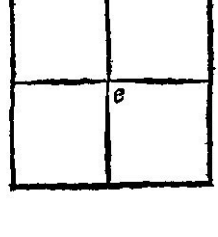
by that center.

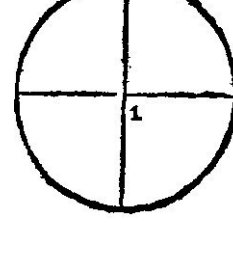
And

8. *It is in the meeting of the diameters.*

As in the examples following. This also followeth out of the same definition of the diameter. For seeing that every diameter passeth by the center: The center must needes be common to all the diameters: and therefore it must also needs be in the meeting of them: Otherwise there should be divers centers of one and the same figure. This also doth the same *Archimedes* propound in the same words in the 8. and 12 theoremes of the same booke, speaking of Parallelogrammes and Triangles.



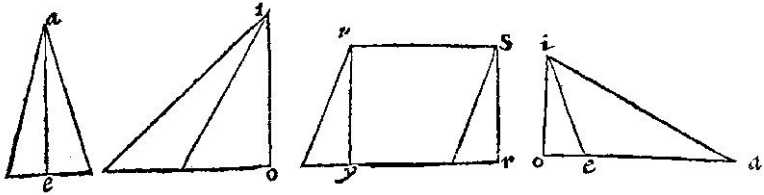




9. *The Altitude is a perpendicular line falling from the toppe of the figure to the base.*

Altitudo, the altitude, or heigth, or the depth: [For that, as hereafter shall bee taught, is but *Altitudo versa*, an heighth with the heeles upward.] As in the figures following are *ae*, *io*, *uy*, or *sr*. Neither is it any matter whether the base be the same with

the figure, or be continued or drawne out longer, as in a blunt angled triangle, when the base is at the blunt corner, as here in the triangle, *aei*, is *ao*.



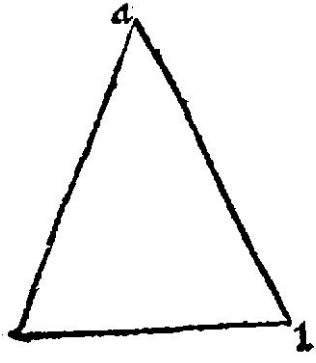
10. *An ordinate figure, is a figure whose bounds are equall and angles equall.*

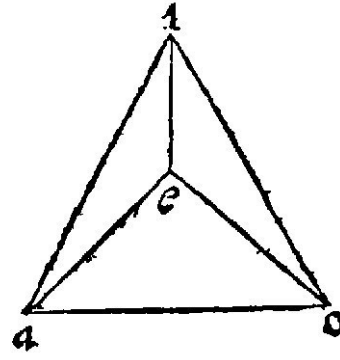
In plaines the Equilater triangle is onely an ordinate figure, the rest are all inordinate: In quadrangles, the Quadrate is ordinate, all other of that sort are inordinate: In every sort of Multangles, or many cornered figures one may be an ordinate. In crooked lined figures the Circle is ordinate, because it is conteined with equall bounds, (one bound alwaies equall to it selfe being taken for infinite many,) because it is equiangled, seeing (although in deede there be in it no angle) the inclination notwithstanding is every where alike and equall, and as it were the angle of the perphery be alwaies alike unto it selfe: whereupon of Plato and Plutarch a circle is said to be *Polygonia*, a multangle; and of Aristotle *Holegonia*, a totangle, nothing else but one whole angle. In mingled-lined figures there is nothing that is ordinate:

In solid bodies, and pyramids the Tetrahedrum is ordinate: Of Prismas, the Cube: of Polyhedrum's, three onely are ordinate, the octahedrum, the Dodecahedrum, and the Icosahedrum. In oblique-lined bodies, the spheare is concluded to bee ordinate, by the same argument that a circle was made to bee ordinate.

11. *A prime or first figure, is a figure which cannot be divided into any other figures more simple then it selfe.*

So in plaines the triangle is a prime figure, because it cannot be divided into any other more simple figure although it may be cut many waies: And in solids, the Pyramis is a first figure: Because it cannot be divided into a more simple solid figure, although it may be divided into an infinite sort of other figures: Of the Triangle all plaines are made; as of a Pyramis all bodies or solids are compounded; such are *aei.* and *aeio.*





12. *A rationall figure is that which is comprehended of a base and height rationall betweene themselves.*

So *Euclide*, at the 1. d. ij. saith, that a rightangled parallelogramme is comprehended of two right lines perpendicular one to another, videlicet one multiplied by the other. For Geometricall comprehension is sometimes as it were in numbers a multiplication: Therefore if yee shall grant the base and height to bee rationalls betweene themselves, that their reason I meane may be expressed by a number of the assigned measure, then the numbers of their sides being multiplyed one by another, the bignesse of the figure shall be expressed. Therefore a Rationall figure is made by the multiplying of two rationall sides betweene themselves.

Therefore,

13. *The number of a rationall figure, is called a Figurate number: And the numbers of which it is made, the Sides of the figurate.*

As if a Right angled parallelogramme be comprehended of the base foure, and the height three, the Rationall made shall be 12. which wee here call the figurate: and 4. and 3. of which it was made, we name sides.

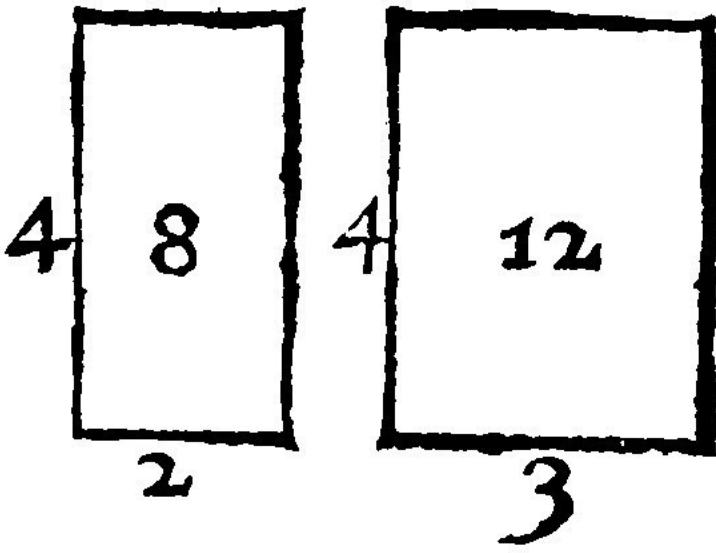
14. *Isoperimetrall figures, are figures of equall perimeter.*

This is nothing else but an interpretation of the Greeke word; So a triangle of 16. foote about, is a isoperimeter to a triangle 16. foote about, to a quadrate 16. foote about, and to a circle 16. foote about.

15. *Of isoperimetralls homogenealls that which is most ordinate, is greatest: Of ordinate isoperimetralls heterogenealls, that is greatest, which hath most bounds.*

So an equilater triangle shall bee greater then an isoperimeter inequilater triangle; and an equicrurall, greater then an unequicrurall: so in quadrangles, the quadrate is greater then that which is not a quadrate: so an oblong more ordinate, is greater then an oblong lesse ordinate. So of those figures which are heterogeneall ordinates, the quadrate is greater then the Triangle: And the Circle, then the Quadrate.

16. *If prime figures be of equall height, they are in reason one unto another, as their bases are: And contrariwise.*



The proportion of first figures is here twofold; the first is direct in those which are of equall height. In Arithmeticke we learned; That if one number doe multiply many numbers, the products shall be proportionall unto the numbers, multiplyed. From hence in rationall figures the content of those which are of equall height is to bee expressed by a number. As in two right angled parallelogrammes, let 4. the same height, multiply 2. and 3. the bases: The products 8. and 12. the parallelogrammes made, are directly proportionall unto the bases 2. and 3. Therefore as 2. is unto 3. so is 8. unto 12. The same shall afterward appeare

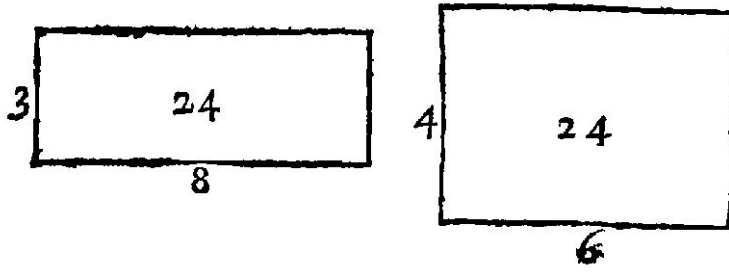
in right Prismes and Cylinders. In plaines, Parallelogramms are the doubles of triangles: In solids, Prismes are the triples of pyramides: Cylinders, the triples of Cones. The converse of this element is plaine out of the former also: First figures if they be in reason one to another as their bases are, then are they of equall height, to witt when their products are proportionall unto the multiplyed, the same number did multiply them.

Therefore,

17. *If prime figures of equall heighth have also equall bases, they are equall.*

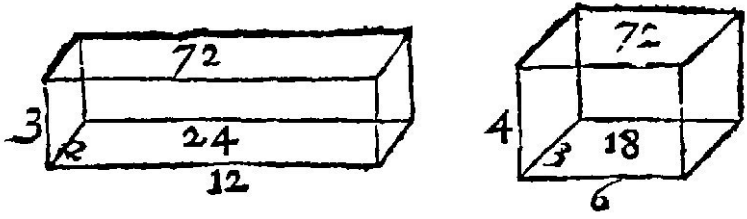
[The reason is, because then those two figures compared, have equall sides, which doe make them equall betweene themselves; For the parts of the one applyed or laid unto the parts of the other, doe fill an equall place, as was taught at the 10. e. j. *Sn.*] So Triangles, so Parallelogrammes, and so other figures proposed are equalled upon an equall base.

18. *If prime figures be reciprocall in base and height, they are equall: And contrariwise.*



The second kind of proportion of first figures is reciprocall. This kinde of proportion rationall and expressible by a number, is not to be had in first figures themselves: but in those that are equally manifold to them, as was taught even now in direct proportion: As for example, Let these two right angled parallelogrammes, unequall in bases and heighths 3, 8, 4, 6, be as heere thou seest: The proportion reciprocall is thus, As 3 the base of the one, is unto 4, the base of the other: so is 6. the height of the one is to 8. the height of the other: And the parallelogrammes are equall, viz. 24. and 24. Againe, let two solids of unequall bases & heights (for here also the base is taken for the length and heighth) be 12, 2, 3, 6, 3, 4. The solids themselves shall be 72. and 72, as here thou seest; and the proportion of the bases and heights likewise is reciprocall: For as 24, is unto 18, so is 4, unto 3. The cause is out of the golden rule of proportion in Arithmeticke: Because twice two sides are proportionall: Therefore the plots made of them shall be

equall. And againe, by the same rule, because the plots are equall: Therefore the bounds are proportionall; which is the converse of this present element.



19. *Like figures are equiangled figures, and proportionall in the shankes of the equall angles.*

First like figures are defined, then are they compared one with another, similitude of figures is not onely of prime figures, and of such as are compounded of prime figures, but generally of all other whatsoever. This similitude consisteth in two things, to witt in the equality of their angles, and proportion of their shankes.

Therefore,

20. *Like figures have answerable bounds subtended against their equall angles: and equall if they themselves be equall.*

Or thus, They have their termes subtended to the equall angles correspondently proportionall: And equall if the figures themselves be equall; H. This is a consectary out of the former definition.

And

21. *Like figures are situate alike, when the proportionall bounds doe answer one another in like situation.*

The second consectary is of situation and place. And this like situation is then said to be when the upper parts of the one figure doe agree with the upper parts of the other, the lower, with the lower, and so the other differences of places. *Sn.*

And

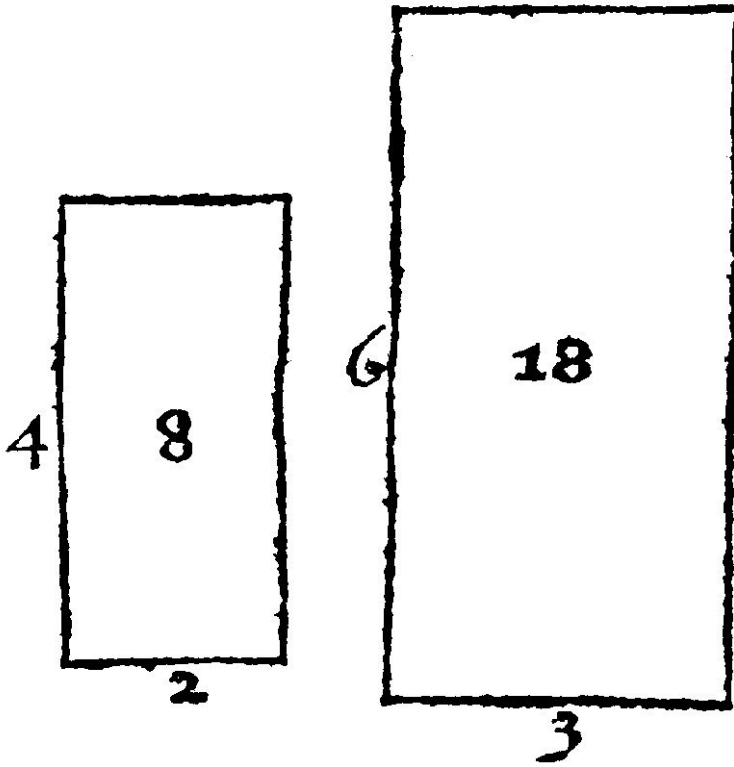
22. *Those figures that are like unto the same, are like betweene themselves.*

This third consectary is manifest out of the definition of like figures. For the similitude of two figures doth conclude both the same equality in angles and proportion of sides betweene themselves.

And

23. *If unto the parts of a figure given, like parts and alike situate, be placed upon a bound given, a like figure and likely situate unto the figure given, shall bee made accordingly.*

This fourth consectary teacheth out of the said definition, the fabricke and manner of making of a figure alike and likely situate unto a figure given. *Sn.*

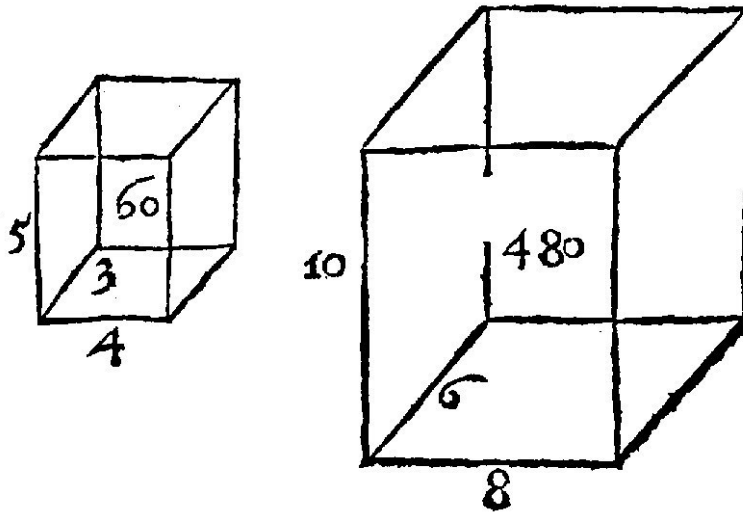


24. *Like figures have a reason of their homologallor correspondent sides equally manifold unto their dimensions: and a meane proportionall lesse by one.*

Plaine figures have but two dimensions, to witt Length, and Breadth: And therefore they have but a doubled reason of their

homologall sides. Solids have three dimensions, videl. Length, Breadth, & thicknesse: therefore they shall have a treabled reason of their homologall or correspondent sides. In 8. and 18. the two plaines given, first the angles are equall: secondly, their homolegall side 2. and 4. and 3. and 6. are proportionall. Therefore the reason of 8. the first figure, unto 18. the second, is as the reason is of 2. unto 3. doubled. But the reason of 2. unto 3. doubled, by the 3. chap. ij. of Arithmeticke, is of 4. to 9. (for 2/3 2/3 is 4/9.) Therefore the reason of 8. unto 18, that is, of the first figure unto the second, is of 4. unto 9. In Triangles, which are the halfes of rightangled parallelogrammes, there is the same truth, and yet by it selfe not rationall and to be expressed by numbers.

Said numbers are alike in the trebled reason of their homologall sides; As for example, 60. and 480. are like solids; and the solids also comprehended in those numbers are like-solids, as here thou seest: Because their sides, 4. 3. 5. and 8. 6. 10. are proportionall betweene themselves. But the reason of 60. to 480. is the reason of 4. to 8. trebled, thus 4/8 4/8 4/8 = 64/512; that is of 1. unto 8. or *octupla*, which you shall finde in the dividing of 480. by 60.



Thus farre of the first part of this element: The second, that like figurs have a meane, proportional lesse by one, then are their dimensions, shall be declared by few words. For plaines having but two dimensions, have but one meane proportionall, solids having three dimensions, have two meane proportionalls. The cause is onely Arithmeticall, as afore. For where the bounds are but 4. as they are in two plaines, there can be found no more but one meane proportionall, as in the former example of 8. and 18. where the homologall or correspondent sides are 2. 3. and 4. 6.

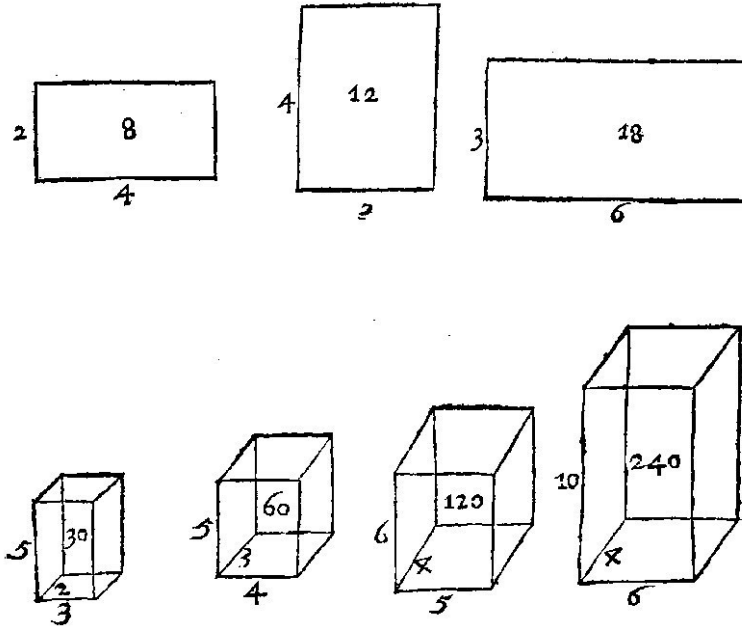
Therefore,

2	3	4	6
	3	4	
8	12		18

Againe by the same rule, where the bounds are 6. as they are in two solids, there may bee found no more but two meane proportionalls: as in the former solids 30. and 240. where the homologall or correspondent sides are 2. 4. 3. 6. 5. 10.

Therefore,

2	4	3	6	5	10
	4	3			
6	12		24		
			24	5	
	30	60	120	240	

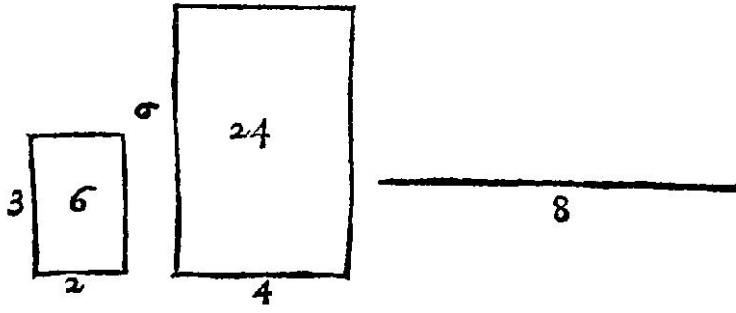


Therefore,

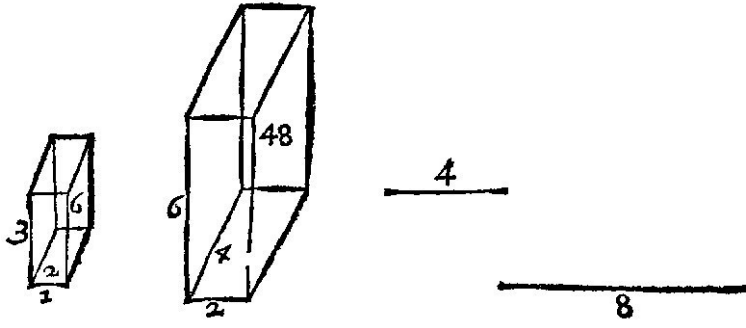
25. *If right lines be continually proportionall, more by one then are the dimensions of like figures likelily situate unto the first and second, it shall be as the first right line is unto the last, so the first figure shall be unto the second: And contrariwise.*

Out of the similitude of figures two consectaries doe arise, in part only, as is their axiome, rationall and expressable by numbers. If three right lines be continually proportionall, it shall be as the first is unto the third: So the rectilineall figure made

upon the first, shall be unto the rectilineall figure made upon the second, alike and likelily situate. This may in some part be conceived and understood by numbers. As for example, Let the lines given, be 2. foot, 4. foote, and 8 foote. And upon the first and second, let there be made like figures, of 6. foote and 24. foote; So I meane, that 2. and 4. be the bases of them. Here as 2. the first line, is unto 8. the third line: So is 6. the first figure, unto 24. the second figure, as here thou seest.



Againe, let foure lines continually proportionall, be 1. 2. 4. 8. And let there bee two like solids made upon the first and second: upon the first, of the sides 1. 3. and 2. let it be 6. Upon the second, of the sides 2. 6. and 4. let it be 48. As the first right line 1. is unto the fourth 8. So is the figure 6. unto the second 48. as is manifest by division. The examples are thus.



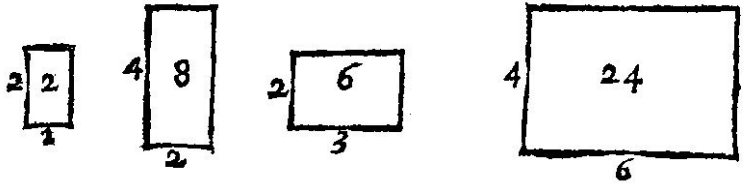
Moreover by this Consectary a way is laid open leading unto the reason of doubling, treabling, or after any manner way whatsoever assigned increasing of a figure given. For as the first right line shall be unto the last: so shall the first figure be unto the second.

And

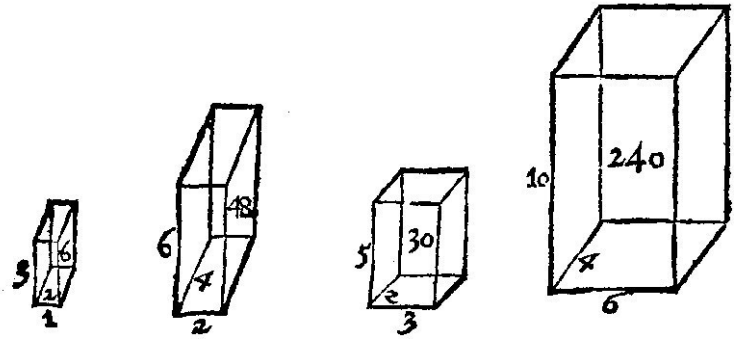
26. *If foure right lines bee proportionall betweene themselves: Like figures likelily situate upon them, shall be also proportionall betweene themselves: And contrariwise, out of the 22. p vj. and 37. p xj.*

The proportion may also here in part bee expressed by numbers: And yet a continuall is not required, as it was in the former.

In Plaines let the first example be, as followeth.



The cause of proportionall figures, for that twice two figures have the same reason doubled.



In Solids let this bee the second example. And yet here the figures are not proportionall unto the right lines, as before figures of equall heighth were unto their bases, but they themselves are proportionall one to another. And yet are they not proportionall in the same kinde of proportion.

The cause also is here the same, that was before: To witt,

because twice two figures have the same reason trebled.

27. *Figures filling a place, are those which being any way set about the same point, doe leave no voide roome.*

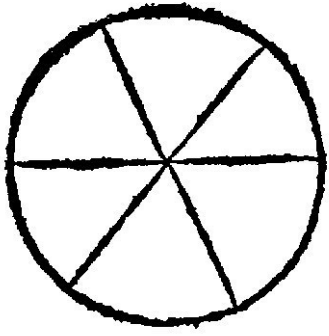
This was the definition of the ancient Geometers, as appeareth out of *Simplicius*, in his commentaries upon the 8. chapter of *Aristotle's* iij. booke of Heaven: which kinde of figures *Aristotle* in the same place deemeth to bee onely ordinate, and yet not all of that kind. But only three among the Plaines, to witt a Triangle, a Quadrate, and a Sexangle: amongst Solids, two; the Pyramis, and the Cube. But if the filling of a place bee judged by right angles, 4. in a Plaine, and 8. in a Solid, the Oblong of plaines, and the Octahedrum of Solids shall (as shall appeare in their places) fill a place; And yet is not this Geometrie of *Aristotle* accurate enough. But right angles doe determine this sentence, and so doth *Euclide* out of the angles demonstrate, That there are onely five ordinate solids; And so doth *Potamon* the Geometer, as *Simplicus* testifieth, demonstrate the question of figures filling a place. Lastly, if figures, by laying of their corners together, doe make in a Plaine 4. right angles, or in a Solid 8. they doe fill a place.

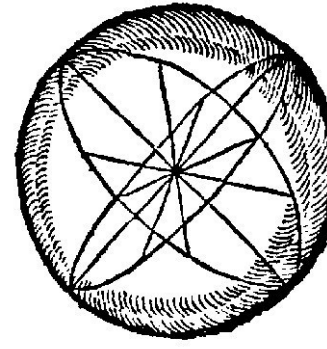
Of this probleme the ancient geometers have written, as we heard even now: And of the latter writers, *Regiomontanus* is said to have written accurately; And of this argument *Maucolycus* hath promised a treatise, neither of which as yet it hath beene our good hap to see.

Neither of these are figures of this nature, as in their due places

shall be proved and demonstrated.

28. *A round figure is that, all whose raies are equall.*





Such in plaines shall the Circle be, in Solids the Globe or Spheare. Now this figure, the Round, I meane, of all Isoperimeters is the greatest, as appeared before at the 15. e. For which cause *Plato*, in his *Timæus* or his Dialogue of the World said; That this figure is of all other the greatest. And therefore God, saith he, did make the world of a sphearicall forme, that within his compasse it might the better containe all things: And *Aristotle*, in his Mechanicall problems, saith; That this figure is the beginning, principle, and cause of all miracles. But those miracles shall in their time God willing, be manifested and showne.

Rotundum, a Roundle, let it be here used for *Rotunda figura*,

a round figure. And in deede *Thomas Finkius* or *Finche*, as we would call him, a learned *Dane*, sequestring this argument from the rest of the body of Geometry, hath intituled that his worke *De Geometria rotundi*, Of the Geometry of the Round or roundle.

29. *The diameters of a roundle are cut in two by equall raies.*

The reason is, because the halfes of the diameters, are the raies. Or because the diameter is nothing else but a doubled ray: Therefore if thou shalt cut off from the diameter so much, as is the radius or ray, it followeth that so much shall still remaine, as thou hast cutte of, to witt one ray, which is the other halfe of the diameter. *Sn*.

And here observe, That *Bisecare*, doth here, and in other places following, signifie to cutte a thing into two equall parts or portions; And so *Bisegmentum*, to be one such portion; And *Bisectio*, such a like cutting or division.

30. *Rounds of equall diameters are equall. Out of the 1. d. iij.*

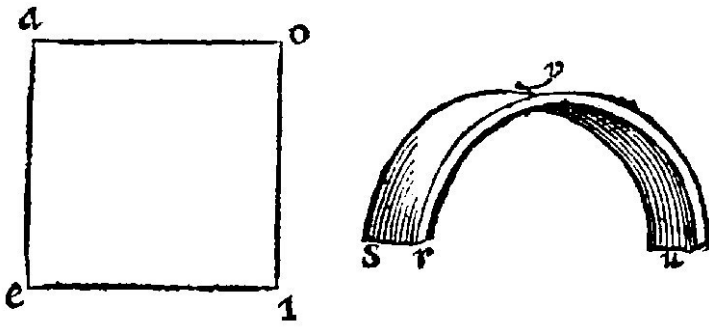
Circles and Spheares are equall, which have equall diameters. For the raies, which doe measure the space betweene the Center and Perimeter, are equall, of which, being doubled, the Diameter doth consist. *Sn*.

The fifth Booke, of *Ramus* his Geometry, which is of Lines and Angles in a plaine Surface

1. *A lineate is either a Surface or a Body.*

Lineatum, (or *Lineamentum*) a magnitude made of lines, as was defined at 1. e. iij. is here divided into two kindes: which is easily conceived out of the said definition there, in which a line is excluded, and a Surface & a body are comprehended. And from hence arose the division of the arte Metriall into Geometry, of a surface, and Stereometry, of a body, after which maner *Plato* in his vij. booke of his Common-wealth, and *Aristotle* in the 7. chapter of the first booke of his *Posteriorums*, doe distinguish betweene Geometry and Stereometry: And yet the name of Geometry is used to signifie the whole arte of measuring in generall.

2. *A Surface is a lineate only broade. 5. d j.*



As here *aeio.* and *uysr.* The definition of a Surface doth comprehend the distance or dimension of a line, to witt Length: But it addeth another distance, that is Breadth. Therefore a Surface is defined by some, as *Proclus* saith, to be a magnitude of two dimensions. But two doe not so specially and so properly define it. Therefore a Surface is better defined, to bee a magnitude onely long and broad. Such, saith *Apollonius*, are the shadowes upon the earth, which doe farre and wide cover the ground and champion fields, and doe not enter into the earth, nor have any manner of thicknesse at all.

Epiphania, the Greeke word, which importeth onely the outter appearance of a thing, is here more significant, because of a Magnitude there is nothing visible or to bee seene, but the surface.

3. *The bound of a surface is a line. 6. d j.*

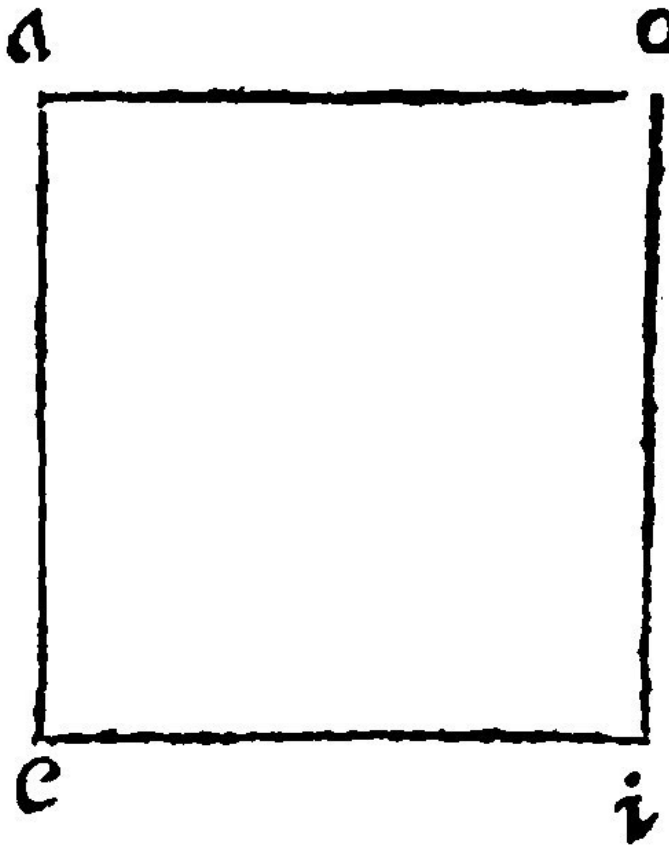
The matter in Plaines is manifest. For a three cornered surface

is bounded with 3. lines: A foure cornered surface, with foure lines, and so forth: A Circle is bounded with one line. But in a Sphearicall surface the matter is not so plaine: For it being whole, seemeth not to be bounded with a line. Yet if the manner of making of a Sphearicall surface, by the conversiō or turning about of a semiperiphery, the beginning of it, as also the end, shalbe a line, to wit a semiperiphery: And as a point doth not only *actu*, or indeede bound and end a line: But is *potentia*, or in power, the middest of it: So also a line boundeth a Surface *actu*, and an innumerable company of lines may be taken or supposed to be throughout the whole surface. A Surface therefore is made by the motion of a line, as a Line was made by the motion of a point.

4. *A Surface is either Plaine or Bowed.*

The difference of a Surface, doth answer to the difference of a Line, in straightnesse and obliquity or crookednesse.

Obliquum, oblique, there signified crooked; Not right or straight: Here, uneven or bowed, either upward or downeward. *Sn.*



5. *A plaine surface is a surface, which lyeth equally betweene his bounds, out of the 7. d j.*

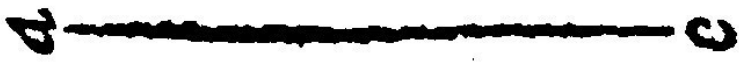
As here thou seest in *aeio*. That therefore a Right line doth looke two contrary waies, a Plaine surface doth looke all about every way, that a plaine surface should, of all surfaces within the same bounds, be the shortest: And that the middest thereof should hinder the sight of the extreames. Lastly, it is equall to the dimension betweene the lines: It may also by one right line every way applyed be tryed, as *Proclus* at this place doth intimate.

Planum, a Plaine, is taken and used for a plaine surface: as before *Rotundum*, a Round, was used for a round figure.

Therefore,

6. *From a point unto a point we may, in a plaine surface, draw a right line, 1 and 2. post. j.*

Three things are from the former ground begg'd: The first is of a Right line. A right line and a periphery were in the ij. booke defined: But the fabricke or making of them both, is here said to bee properly in a plaine.



The fabricke or construction of a right line is the 1. petition. And justly is it required that it may bee done onely upon a plaine: For in any other surface it were in vaine to aske it. For

neither may wee possibly in a sphericall betweene two points draw a right line: Neither may wee possibly in a Conicall and Cylindraceall betweene any two points assigned draw a right line. For from the toppe unto the base that in these is only possible: And then is it the bounde of the plaine which cutteth the Cone and Cylinder. Therefore, as I said, of a right plaine it may onely justly bee demanded: That from any point assigned, unto any point assigned, a right line may be drawne, as here from *a* unto *e*.

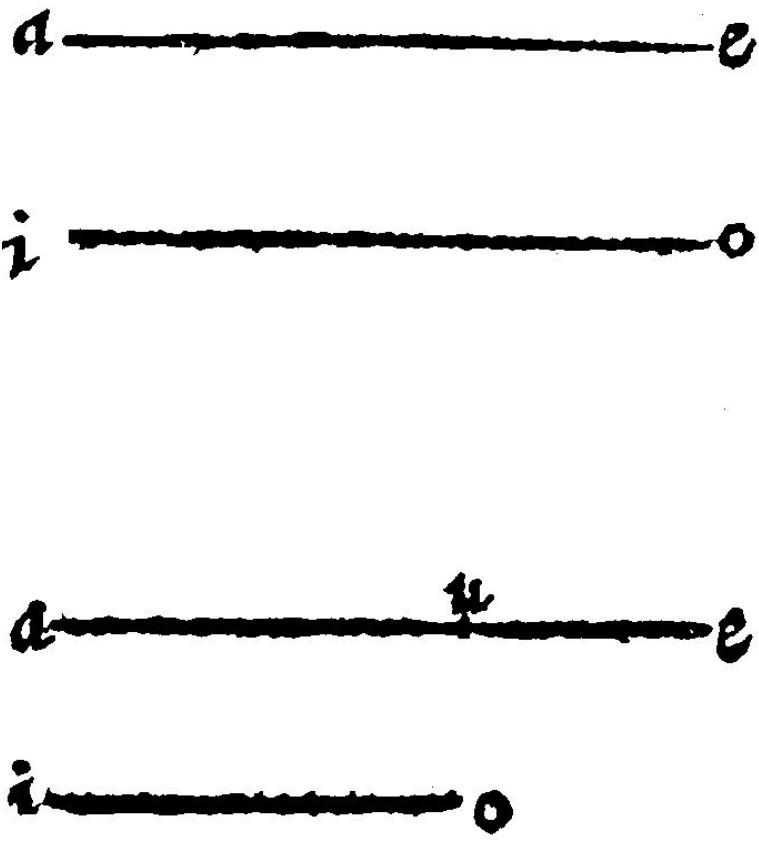
Now the Geometricall instrument for the drawing of a right plaine is called *Amussis*, & by *Petolemey*, in the 2. chapter of his first booke of his Musicke, *Regula*, a Rular, such as heere thou seest.

And from a point unto a point is this justly demanded to be done, not unto points; For neither doe all points fall in a right line: But many doe fall out to be in a crooked line. And in a Spheare, a Cone & Cylinder, a Ruler may be applyed, but it must be a sphearicall, Conicall, or Cylindraceall. But by the example of a right line doth *Vitellio*, 2 *p j.* demaund that betweene two lines a surface may be extended: And so may it seeme in the Elements, of many figures both plaine and solids, by *Euclide* to be demanded; That a figure may be described, at the 7. and 8. e

ij. Item that a figure may be made vp, at the 8. 14. 16. 23. 28. p. vj. which are of Plaines. Item at the 25. 31. 33. 34. 36. 49. p. xj. which are of Solids. Yet notwithstanding a plaine surface, and a plaine body doe measure their rectitude by a right line, so that *jus postulandi*, this right of begging to have a thing granted may seeme primarily to bee in a right plaine line.

Now the *Continuation* of a right line is nothing else, but the drawing out farther of a line now drawne, and that from a point unto a point, as we may continue the right line *ae.* unto *i.* wherefore the first and second Petitions of *Euclide* do agree in one.

And



7. *To set at a point assigned a Right line equall to another right line given: And from a greater, to cut off a part equall to a lesser. 2. and 3. p j.*

As let the Right line given be *ae.* And to *i.* a point assigned,

grant that *io.* equall to the same *ae.* may bee set. Item, in the second example, let *ae.* bee greater then *io.* And let there be cut off from the same *ae.* by applying of a rular made equall to *io.* the lesser, portion *au.* as here. For if any man shall thinke that this ought only to be don in the minde, hee also, as it were, beares a ruler in his minde, that he may doe it by the helpe of the ruler. Neither is the fabricke in deede, or making of one right equal to another: And the cutting off from greater Right line, a portion equall to a lesser, any whit harder, then it was, having a point and a distance given, to describe a circle: Then having a Triangle, Parallelogramme, and semicircle given, to describe or make a Cone, Cylinder, and spheare, all which notwithstanding *Euclide* did account as principles.

Therefore,

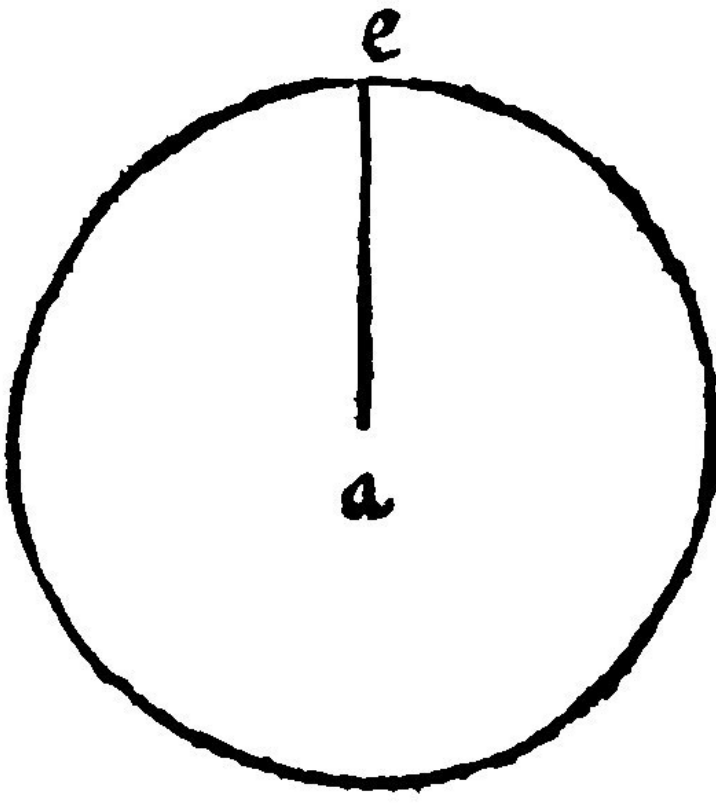
8. *One right line, or two cutting one another, are in the same plaine, out of the 1. and 2. p xj.*

One Right line may bee the common section of two plaines: yet all or the whole in the same plaine is one: And all the whole is in the same other: And so the whole is the same plaine. Two Right lines cutting one another, may bee in two plaines cutting one of another; But then a plaine may be drawne by them: Therefore both of them shall be in the same plaine. And this plaine is geometrically to be conceived: Because the same plaine is not alwaies made the ground whereupon one oblique line, or two cutting one another are drawne, when a periphery is in a sphearicall: Neither may all peripheries cutting one another be

possibly in one plaine.

And

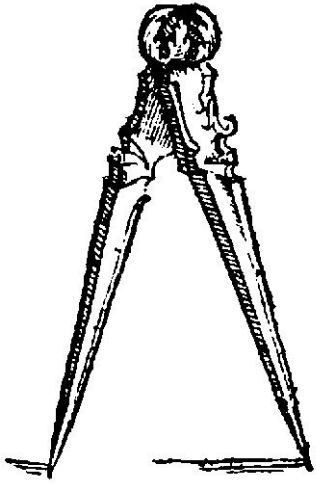
9. *With a right line given to describe a peripherie.*

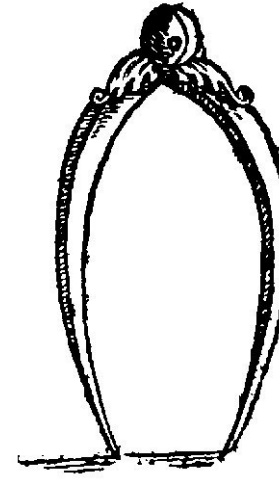


This fabricke or construction is taken out of the 3. Petition which is thus. Having a center and a distance given to describe, make, or draw a circle. But here the terme or end of a circle is onely sought, which is better drawne out of the definition of a periphery, at the 10. e ij. And in a plaine onely may that conversion or turning about of a right line bee made: Not in a sphearicall, not in a Conicall, not in a Cylindraceall, except it be in top, where notwithstanding a periphery may bee described. Therefore before (to witt at the said 10. e ij.) was taught the generall fabricke or making of a Periphery: Here we are informed how to discribe a Plaine periphery, as here.

Now as the Rular was the instrument invented and used for the drawing of a right line: so also may the same *Rular*, used after another manner, be the instrument to describe or draw a periphery withall. And indeed such is that instrument used by the Coopers (and other like artists) for the rounding of their bottomes of their tubs, heads of barrells and otherlike vessells: But the *Compasses*, whether straight shanked or bow-legg'd, such as here thou seest, it skilleth not, are for al purposes and practises, in this case the best and readiest. And in deed the Compasses, of all geometricall instruments, are the most excellent, and by whose help famous Geometers have taught: That all the problems of geometry may bee wrought and performed: And there is a booke extant, set out by *John Baptist*, an Italian, teaching, How by one opening of the Compasses all the problems of *Euclide* may be resolved: And *Jeronymus Cardanus*, a famous

Mathematician, in the 15. booke of his Subtilties, writeth, that there was by the helpe of the Compasses a demonstration of all things demonstrated by *Euclide*, found out by him and one *Ferrarius*.



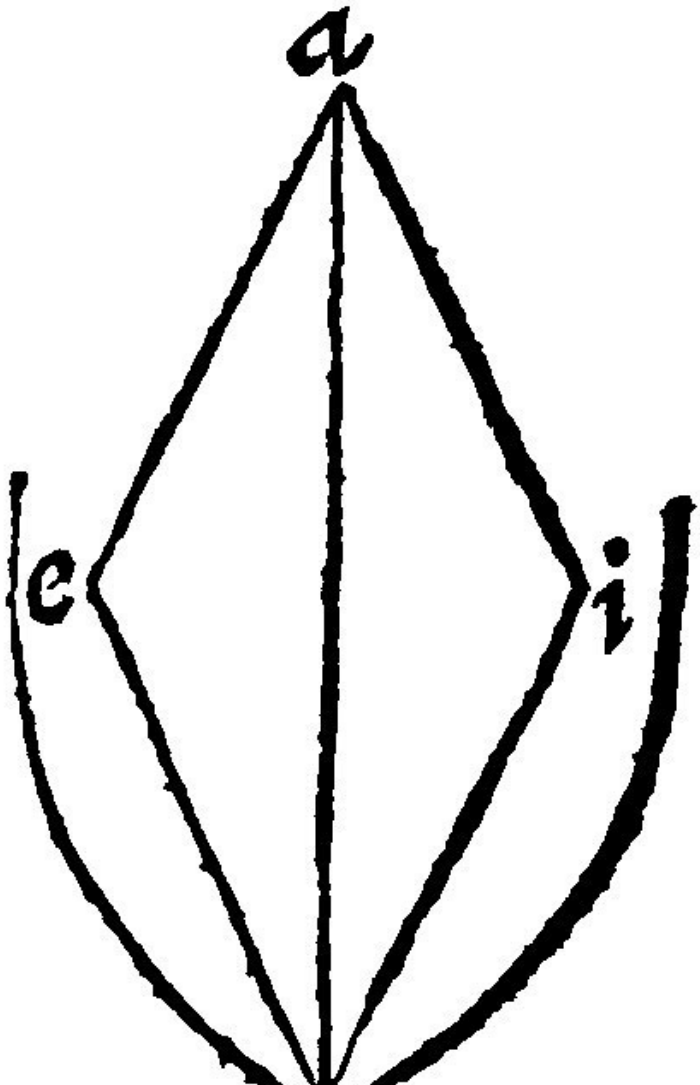


Talus, the nephew of *Dædalus* by his sister, is said in the viij. booke of *Ovids Metamorphosis*, to have beene the inventour of this instrument: For there he thus writeth of him and this matter: *—Et ex uno duo ferrea brachia nodo: Iunxit, ut æquali spatio distantibus ipsis: Altera pars staret, pars altera duceret orbem.*

Therfore

10. *The raies of the same, or of an equall periphery, are equall.*

The reason is, because the same right line is every where converted or turned about. But here by the Ray of the periphery, must bee understood the Ray the figure contained within the periphery.

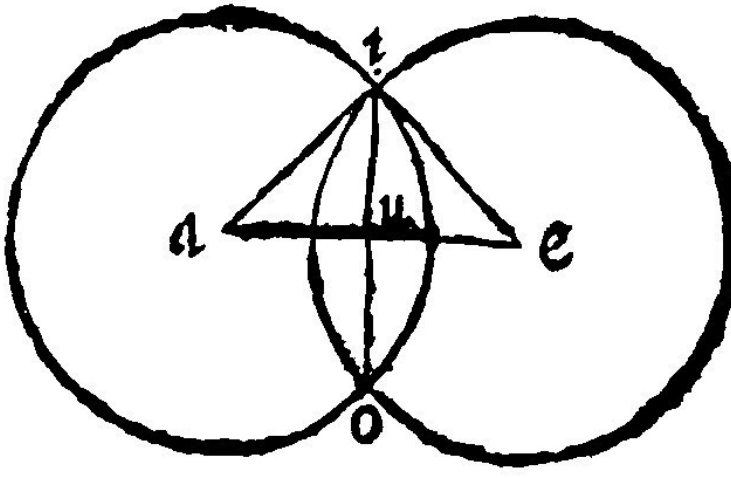

a
e
i

11. *If two equall peripheries, from the ends of equall shankes of an assigned rectilineall angle, doe meete before it, a right line drawne from the meeting of them unto the toppe or point of the angle, shall cut it into two equall parts. 9. p j.*

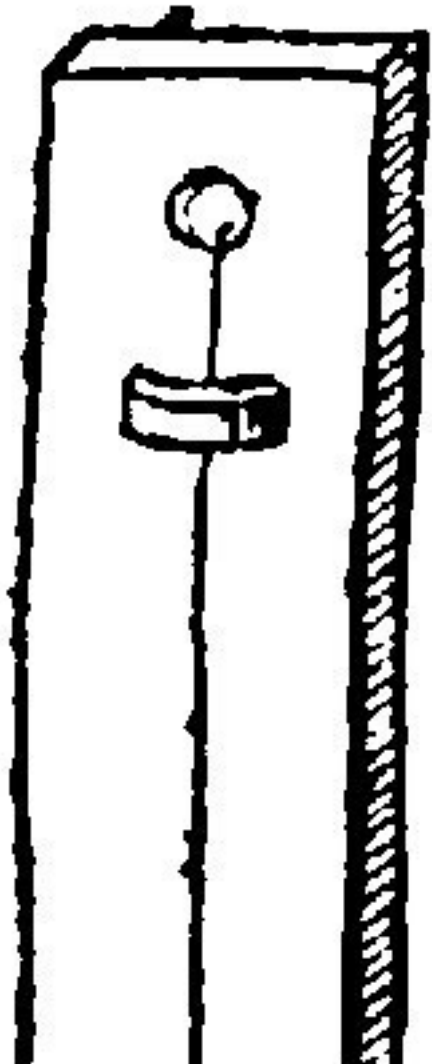
Hitherto we have spoken of plaine lines: Their affection followeth, and first in the Bisection or dividing of an Angle into two equall parts.

Let the right lined Angle to bee divided into two equall parts bee *eai.* whose equall shankes let them be *ae.* and *ai.* (or if they be unequall, let them be made equall, by the 7 e.) Then two equall peripheries from the ends *e* and *i.* meet before the Angle in *o*. Lastly, draw a line from *o.* unto *a*. I say the angle given is divided into two equall parts. For by drawing the right lines *oe.* and *oi.* the angles *oae.* and *oai.* equicrurall, by the grant, and by their common side *ao.* are equall in base *eo.* and *io.* by the 10 e (Because they are the raies of equall peripheries.) Therefore by the 7. e iij. the angles *oae.* and *oai.* are equall: And therefore the Angle *eai.* is equally divided into two parts.

12. *If two equall peripheries from the ends of a right line given, doe meete on each side of the same, a right line drawne from those meetings, shall divide the right line given into two equall parts. 10. p j.*



Let the right line given bee *ae.* And let two equall peripheries from the ends *a.* and *e.* meete in *i.* and *o.* Then from those meetings let the right line *io.* be drawne. I say, That *ae.* is divided into two equall parts, by the said line thus drawne. For by drawing the raies of the equall peripheries *ia.* and *ie.* the said *io.* doth cut the angle *aie.* into two equall parts, by the 11. e. Therefore the angles *aiu.* and *uie.* being equall and equicrurall (seeing the shankes are the raies of equall peripheries, by the grant.) have equall bases *au.* and *ue.* by the 7. e iij. Wherefore seeing the parts *au.* and *ue.* are equall, *ae.* the assigned right line is divided into two equall portions.



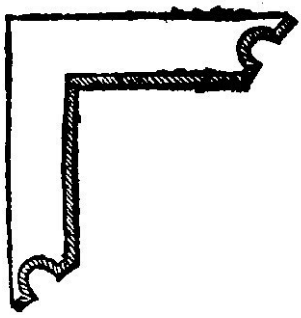
13. *If a right line doe stand perpendicular upon another right line, it maketh on each side right angles: And contrary wise.*

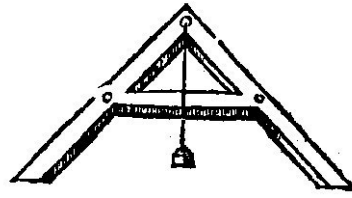
A right line standeth upon a right line, which cutteth, and is not cut againe. And the *Angles on each side*, are they which the falling line maketh with that underneath it, as is manifest out of *Proclus*, at the 15. pj. of *Euclide*; As here *ae.* the line cut: and *io.* the insisting line, let them be perpendicular; The angles on each side, to witt *aio.* and *eio.* shall bee right angles, by the 13. e iij.

The *Rular*, for the making of straight lines on a plaine, was the first Geometricall instrument: The *Compasses*, for the describing of a Circle, was the second: The *Norma* or *Square* for the true erecting of a right line in the same plaine upon another right line, and then of a surface and body, upon a surface or body, is the third. The figure therefore is thus.

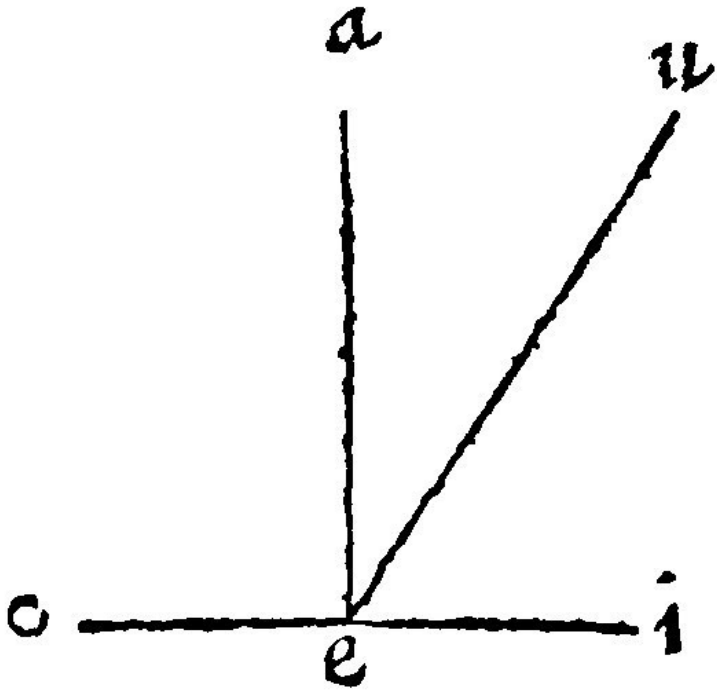
Now *Perpendiculū*, an instrument with a line & a plummet of leade appendant upon it, used of Architects, Carpenters, and Masons, is meerely physicall: because heavie things naturally by their weight are in straight lines carried perpendicularly downeward. This instrument is of two sorts: The first, which they call a Plumbe-rule, is for the trying of an erect perpendicular, as whether a columne, pillar, or any other kinde of building bee right, that is plumbe unto the plaine of the horizont & doth not leane or reele any way. The second is for the trying or examining of a plaine or floore, whether it doe lye parallell to the horizont or not. Therefore when the line from the right angle, doth fall upon

the middle of the base; it shall shew that the length is equally poysed. The Latines call it *Libra*, or *Libella*, a ballance: of the *Italians Livello*, and vel *Archipendolo*, *Achildulo*: of the *French*, *Nivelle*, or *Niueau*: of us a *Levill*.





Therefore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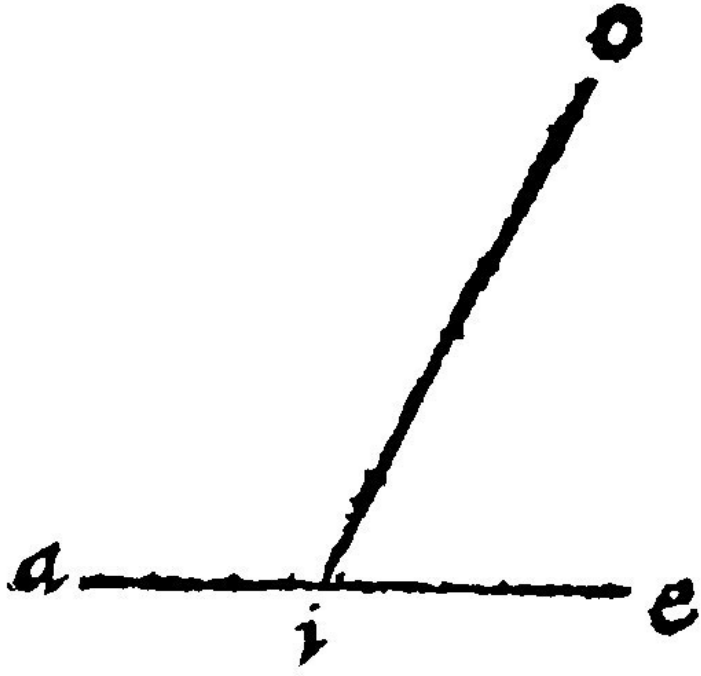

14. *If a right line do stand upon a right line, it maketh the angles on each side equall to two right angles: and contrariwise out of the 13. and* 14. *p j.*

For two such angles doe occupy or fill the same place that two right angles doe: Therefore they are equall to them by the 11. e j. If the insisting line be perpendicular unto that underneath it, it then shall make 2. right angles, by the 13. e. If it bee not

perpendicular, & do make two oblique angles, as here *aio.* and *oie.* are yet shall they occupy the same place that two right angles doe: And therefore they are equall to two right angles, by the same.

The converse is forced by an argument *ab impossibli*, or *ab absurdo*, from the absurdity which otherwise would follow of it: For the part must otherwise needes bee equall to the whole. Let therefore the insisting or standing line which maketh two angles *aeo.* and *aeu.* on each side equall to two right angles, be *ae.* I say that *oe.* and *ei.* are but one right line. Otherwise let *oe.* bee continued unto *u.* by the 6. e. Now by the 14. e. or next former element, *aeo.* & *aeu.* are equall to two right angles; To which also *oea.* & *aei.* are equall by the grant: Let *aeo.* the common angle be taken away: then shall there be left *aeu.* equall to *aei.* the part to the whole, which is absurd and impossible. Herehence is it certaine that the two right lines *oe,* and *ei,* are in deede but one continuall right line.

Конец ознакомительного фрагмента.