$$a^n + b^n = c^n$$

Youri Kraskov

The Wonders of Arithmetic From Pierre Simon de Fermat

$$A^X + B^Y = C^Z$$

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«ЛитРес: Самиздат»

2021

Kraskov Y. V.

The Wonders of Arithmetic from Pierre Simon de Fermat /
Y. V. Kraskov — «ЛитРес: Самиздат», 2021

ISBN 978-5-532-98629-9

This book shows how the famous scientific problem called "Fermat Last theorem" (FLT) allows us to reveal the insolvency and incapacity of science, in which arithmetic for various historical reasons has lost the status of the primary basis of all knowledge. The unusual genre of the book was called "Scientific Blockbuster", what means a combination of an action-packed narrative in the style of fiction with individual fragments of purely scientific content. The original Russian text of this book is translated into English by its author Youri Kraskov.

ISBN 978-5-532-98629-9

Содержание

Youri Kraskov
The Wonders of Arithmetic from Pierre Simon de Fermat

Abstract

Within the framework of the designated topic:

1. There are restored some little-known facts from the Pierre Fermat's biography.

2. For the first time, there are given *two definitions (mathematical and general) the concept of number*, as well as a *new versions axioms and basic properties of numbers* following from them.

3. There is shown fallacy of Euclid, Gauss, and Zermelo's proofs of the Basic Theorem of Arithmetic (BTA), without which the foundation of the whole science collapses.

4. There are given comments on Zermelo's proof of the Basic Theorem of Arithmetic.

5. There is restored the *Fermat's BTA proof by the descent method*.

6. There is restored a simplest method of proving the FLT for the 4th power.

7. There is shown the fallacy of the G. Frey's idea lied in the basis of the A. Wiles' FLT proof 1995.

8. There is restored the *Fermat's proof of his Last Theorem* on the basis of a new way to solving the Pythagoras' equation $x^2+y^2=z^2$ by using the formula, discovered by him and called *Fermat Binomial*.

9. As a consequence of the FLT proof there are formulated *Theorems on Magic Numbers*, the validity of which is confirmed by examples of calculations.

10. There is proposed the formulation of the *Beal Theorem* revealing the essence of the *Beal Conjecture* for equation $A^x+B^y=C^z$. There are given examples of calculations according to this theorem.

11. There is restored a way for proving the *Fermat's Golden Theorem*.

12. There is restored the proof's method of the Fermat's grandiose discovery about primes of the form 4n+1.

13. There is restored a way of solving the Archimedes-Fermat equation $Ax^2+1=y^2$.

14. There is restored a proof of the Fermat's theorem on the unique solution of the equation $y^3=x^2+2$

15. There are shown examples to application of methods for solving problems proposed by Fermat.

16. There is shown a role of arithmetic as the basis of foundations of the whole science.

17. There are shown examples of non-existent sciences such as history, informatics and economics.

18. There are given *definitions the essence of the basic concepts' informatics and economics*.

19. For the first time, there is given a *general definition the essence of the concept an information*.

20. There are proposed some *fundamentals of informatics as a science*.

21. There is proposed a method for *ordering knowledge* using the *Basic law of systems*.

22. There is proposed the idea of an *economic breakthrough* based on the *new generation of IT*.

23. There is proposed *the new essential understanding of money and their functions*.

24. There is proposed an *International Payment System (IPS) of a fundamentally new type* using national currencies in international settlements.

25. There is given essential understanding the *source of profit on invested capital*.

26. Historical episodes are presented in a stylized literary form.
27. There is proposed the *restoration of the tombstone of P. Fermat* with an English translation.
28. There are proposed 15 riddles in Fermat-style i.e. without their complete solution.
29. There is compiled a full list of scientific problems presented in this book in 100 points.
30. The original Russian text of the book is translated by its author into English.

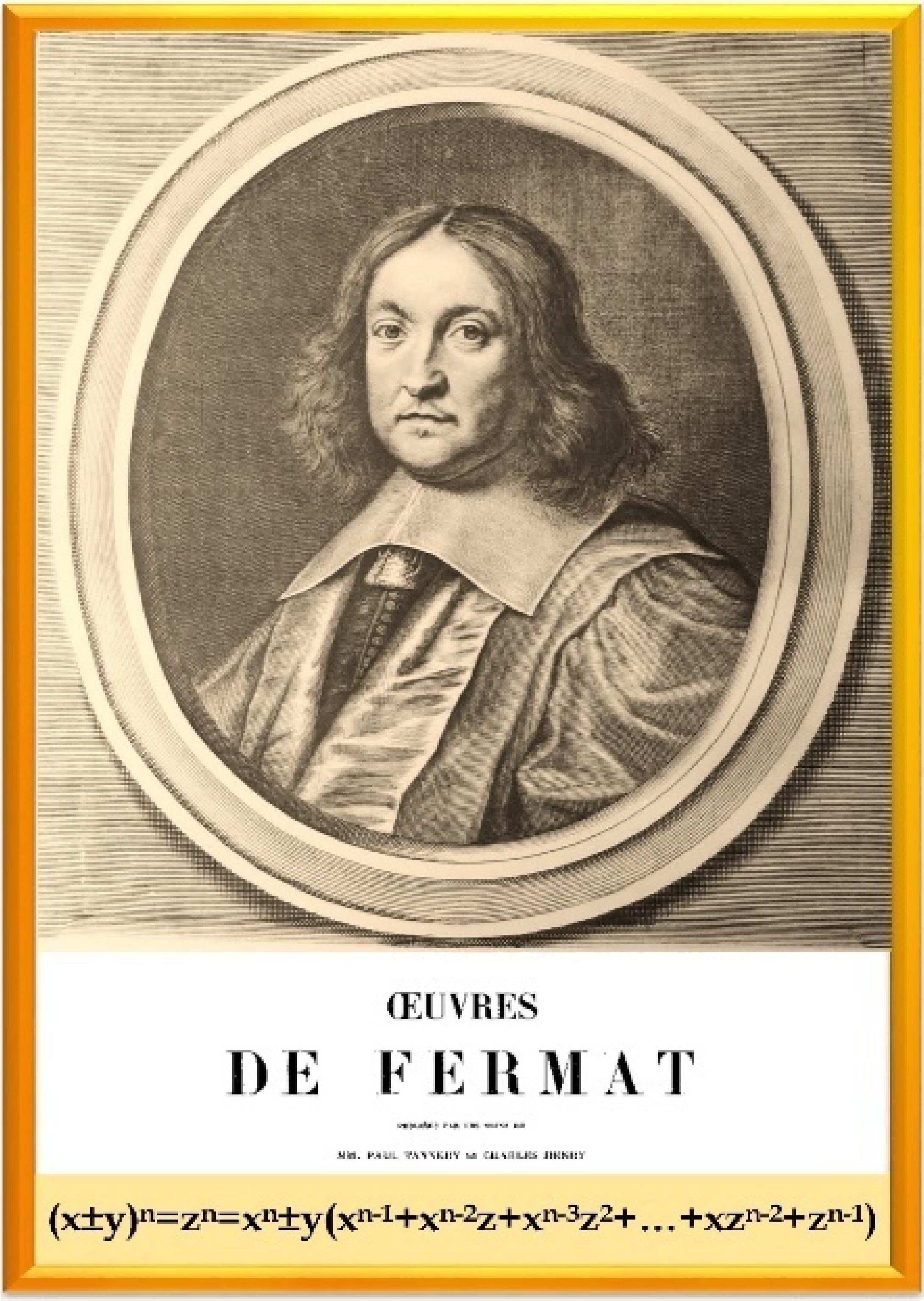
The book is intended for a wide range of readers.

Feedback and comments may be sent to ***c_city2000@mail.ru***

From the Author

Imagine that you decided to write a book and such an interesting one so that, wow, it is breathtaking! But how to begin? It's very simple, you open this book and immediately see the famous picture with the portrait of our main hero the legendary Pierre de Fermat from the complete collection of all his written works, that Paul Tannery and Charles Henry released at the beginning of the 20th century. So here this picture is now decorated with a wonderful equation called Fermat’s binomial formula, about which the current science still has no idea, although this formula was published back in 2008.

Pic. 1. Portrait of the Senator Pierre de Fermat



And the next picture is a delightful sculptural composition. Our hero is so good-looking and next to him is his muse, from her he drew his inspiration and created such wonders, from which the entire civilized world since the XVII century and until now just goes crazy.

Pic. 2. *Pierre Fermat and His Muse* in the Capitol of Toulouse

After this picture follows the image of a page from Diophantus' "Arithmetic" with the text of the Fermat's Last Theorem, published in 1670, and from here immediately the main theme appears.

Pic. 3. Diophantus's "Arithmetic" Page of the Greek-Latin Edition 1670 with the Task VIII and a Remark to it, Becoming Later the *Fermat's Last Theorem*


Arithmeticorum Liber II. 61

IN QVAESTIONEM VII.

QVAESTIO VIII.

OBSERVATIO DOMINI PETRI DE FERMAT.

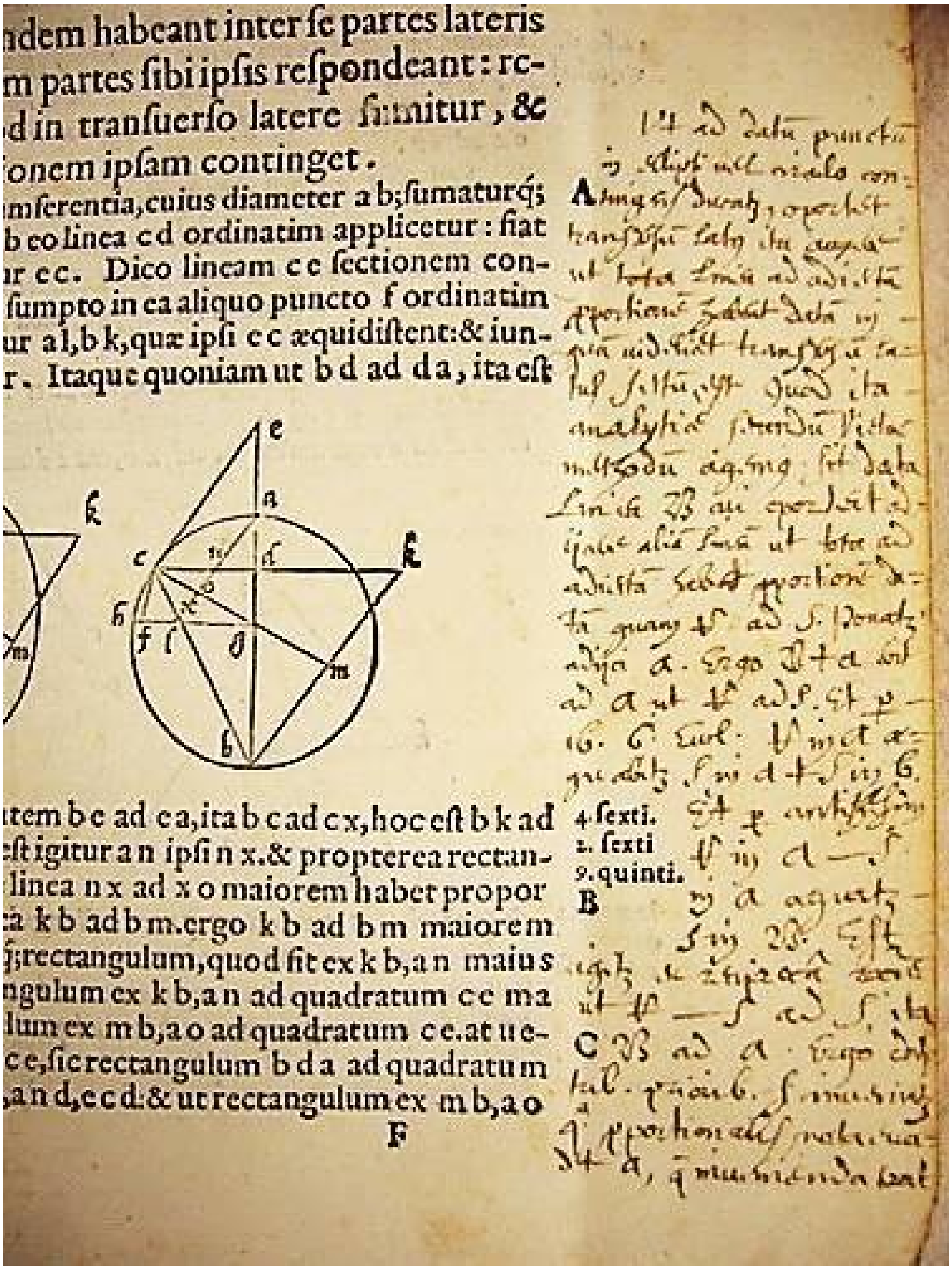
Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est diuidere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

QVAESTIO IX.

But this is about science and can it really be something interesting?

In the one, that we are taught, it is hardly, but in the true science there can be really amazing wonders! Because, unlike conventional belles-lettres, science is not just a literary embodiment of the author's idea, everything here is much more complicated, since he should reflect not invented by him, but the truest reality, which can always be checked and if something is wrong, then the whole work will go down the drain. Such are the cruel laws of this genre. However, really, if you just look at the content of the book … there are some problems, tasks, refutations … outwardly it looks somehow not very intriguing. But the impression will change when we after the two wonderful pictures at once take the bull by the horns – we give another picture by our hero with his really written marks in the margins of one old book …

Pic. 4. Fermat's Recording in Margins of *Apollonius the Perganos*' Book, named *Conic Section*


ndem habeant inter se partes lateris
m partes sibi ipsis respondeant: re-
d in transuerso latere sumitur, &
ionem ipsam continget.
mferentia, cuius diameter a b; sumaturq;
b eo linea c d ordinatim applicetur: fiat
ir c c. Dico lineam c e sectionem con-
sumpto in ea aliquo puncto f ordinatim
ur a l, b k, quæ ipsi c e æquidistent: & iun-
r. Itaque quoniam ut b d ad d a, ita est

item b c ad c a, ita b c ad c x, hoc est b k ad
est igitur a n ipsi n x. & propterea rectan-
linea n x ad x o maiorem habet propor
a k b ad b m. ergo k b ad b m maiorem
q; rectangulum, quod fit ex k b, a n maius
ngulum ex k b, a n ad quadratum c e ma
lum ex m b, a o ad quadratum c e. at ue-
c e, sic rectangulum b d a ad quadratum
, a n d, e c d: & ut rectangulum ex m b, a o
F

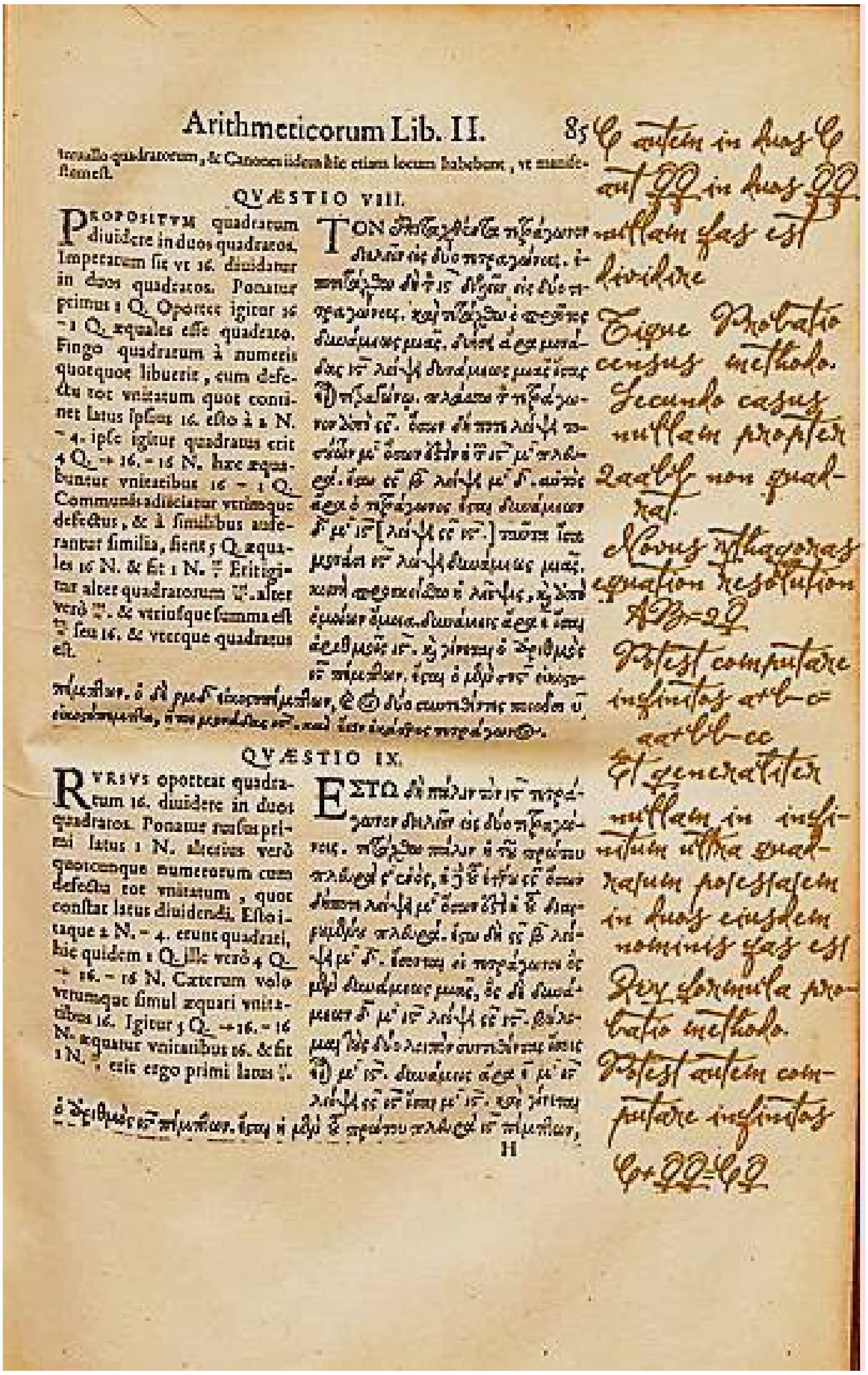
A

4. sexti.
2. sexti
9. quinti.
B

C

and right behind it suddenly … bubuh!!!

Pic. 5. The Page of the Diophantus' "Arithmetic" Edition 1621 and the Restored Text of the *Fermat Last Theorem* in the Margins



Arithmeticorum Lib. II. 85

[illegible] quadratorum, & Canones iidem hic etiam locum habebunt, vt manifestum est.

QVÆSTIO VIII.

PROPOSITVM quadratum diuidere in duos quadratos. Imperatum sit vt 16. diuidatur in duos quadratos. Ponatur primus 1 Q. Oportet igitur 16 − 1 Q. æquales esse quadrato. Fingo quadratum à numeris quotquot libuerit, cum defectu tot vnitatum quot continet latus ipsius 16. esto à 2 N. − 4. ipse igitur quadratus erit 4 Q. + 16. − 16 N. hæc æquabuntur vnitatibus 16 − 1 Q. Communis adiiciatur vtrimque defectus, & à similibus auferantur similia, fient 5 Q. æquales 16 N. & fit 1 N. 16/5. Erit igitur alter quadratorum 256/25. alter verò 144/25. & vtriusque summa est 400/25. seu 16. & vterque quadratus est.

QVÆSTIO IX.

RVRSVS oporteat quadratum 16. diuidere in duos quadratos. Ponatur rursus primi latus 1 N. alterius verò quotcunque numerorum cum defectu tot vnitatum, quot constat latus diuidendi. Esto itaque 2 N. − 4. erunt quadrati, hic quidem 1 Q. ille verò 4 Q. + 16. − 16 N. Cæterum volo vtrumque simul æquari vnitatibus 16. Igitur 5 Q. + 16. − 16 N. æquatur vnitatibus 16. & fit 1 N. [illegible] erit ergo primi latus [illegible].

H

C autem in duos C aut QQ in duos QQ nullam fas est dividere

Cique Probatio census methodo.

Secundo casus nullam propter 2aab non quadrat.

Novus Pythagoras equation resolution A2B=2Q

Potest computare infinitas a+b-c a+b-c

Et generatiter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est

Qui formula probatio methodo.

Potest autem computare infinitas C+QQ=CQ

Oh, what is it??? … This is just not possible!!! Even eyes cannot believe. A page from Diophantus' "Arithmetic" only an earlier edition 1621 from Fermat's personal copy of the book with the most famous theorem on the margins, from which the entire scientific world is still in a fever! Everyone knows this book has been lost! But where does it come from?

From there! The book was so well hidden that so far no one knew about it!

Yeah … and what's next?

The next is that Fermat's binomial formula is from this book of ours, where it will be derived, and it is easily to make sure that formula is true. Take any two numbers x, y and the third z their sum (or difference), substitute in the formula and everything will be OK! Isn't that curious?

Well, but what is the avail of this formula and what is its significance?

But without it the Fermat Last Theorem will remain unproven!

Yeah… would Fermat really hide such a treasure from everyone?

Well, that's a good question. Anyway, the plot is already twisted in the right direction and quite famously … although this is not even the beginning and only a light aperitif, but it acting already is oh so good!

Is it true that a lot of money was promised for this theorem?

Hmm, promised – yes, and even paid … but not at all for the FLT!

How is this so, and for what?

For the error, that was revealed in the proof, but science did not want notice it for 25 years.

So, it turns out all this work with our book will go past the cash box?

Well, that's another good question. However, it has already long been known that for science money is not paid. Well, if … as if for … doing science … or taking care of it … well, for the good of the state, then yes, billions are spinning here, but the very science has never existed and does not exist now.

So perhaps we need to do something incredible here in this book so that we can get something out of these billions?

But no, then no one will read this book, besides all the places of the distribution of money have long been occupied. It is necessary to do this book with real science, but for this we need to use special approaches and principles. Particularly, the main principle should be that all the topics touched upon here contain secrets, and such that they are hidden not otherwise … as it is said … ah yes, behind seven seals!

Wow, it's light to say…

Moreover, about this should be announced in advance (we are now doing this) that even in relatively calm places of narration no one should have any doubt that in every topic something must necessarily be very much surprise or even shake! Well, not literally, of course. Say, it is given in some place a calm explanation of some entity or their components and suddenly, unexpectedly … bubuh!!! And it turns out that here the amazing secret is hiding, which was lurking under the guise of what has been known for a long time, but did not attract any attention. Another principle is that of all the secrets we have revealed (and there are more than a hundred of them here!!!) we must leave some share (say, one and half dozen) as undisclosed. Otherwise the children will be offended that them no one nothing left for their own discoveries …

!!!… What?… The children???

Oh yes, we narrowly forgot to say that this book is largely childish. Most adults would be unable to retrain for a such a thorough science. It is clear, they will very offended hearing this, but we of course will not trumpet about it over the entire world, and even will observe decency i.e. from all the tasks of basic science that we will discuss, only the lightest will be singled out specifically for children. They'll be happy about this because among these tasks there will be … the very FLT!!! Yes, it is in this book of ours that for the first time in the entire history of the FLT, this problem, and not just a problem, but one of the fundamental basics of science (!!!) will be solved in expanded form together with some other problems related to it. In this sense, the FLT will be our guiding star and its first-discoverer Pierre Fermat will be our mentor from whom we will directly (!) receive all the information we need.

However, the FLT is only one of a whole hundred (!!!) tasks that will also receive solutions here for the first time! Along with the solution of the FLT problem, there will be a whole heap of other very impressive tasks that will delight many kids and interest them for a long time. And we will try very hard to get their attention, then they also will draw adults into this game. If we compare the significance of the tasks solved here, the undisputed leadership will remain by the FLT although as for the difficulty of its solving, it is quite simple and secondary school students can easily cope with it … Do you doubt? But still this is a task from the 17th century, however today's ones will be more difficult, but in order to solve them, you need to have a reliable foundation of knowledge and FLT is one of its cornerstones.

About other tasks we will else tell in detail and they will then cease to be a secret behind seven seals, but those that we only call and will not reveal to the end, will be very curious. Take for example one problem of astronomers who have accumulated huge collections with records of cosmic signals of electromagnetic radiation from all points of the starry sky. And what to do with them nobody knows. Whether it is the brothers in mind communicate with us or it is just natural radiation. Unmeasured money spent on giant radio telescopes and a bunch of people working for them, but there’s nothing to use. They are simple the poor fellows!

But we know a way how to distinguish reasonable signals from natural ones, but we will lowly keep silent about this. We know this only because we will reveal here one of the greatest science’s secrets about determining the essence of the information’s phenomenon. It is clear the current science in this matter doesn’t know anything at all. But as soon as we clarify it, then besides us also others can then guess how electromagnetic signals may be analyzed for determining their reasonable origin.

We'll begin to approach the solution of this problem by penetrating the most important problem of all science, this is defining the essence of the number phenomenon, about which unfortunately science also does not know anything like about many other “simple” questions. And now, when it comes to the most important essence capable of displaying all other essences (yes, yes, it's about number!), we can already see that case is taking a much more serious turn and such shifts are awaiting us, which were not even in the entire visible history of our civilization!!!

Again, are you in doubt? But that is why you would never guess what are those shifts, and when you find out, you can hardly believe that they are possible, but secrets behind seven seals cannot be other. Until someone gets to them and begins to slowly reveal them, everyone will remain in the dark about how a much is arranged in this world and why it is better to know about it than to live in ignorance about the existence of the wonderful world of science where perfection and precision dominate. And it is just curious to find out how, for example, does look the proofs of the Fermat’s theorems that turned out to be inaccessible even to the greatest scientists. Now there is a book that allows everyone to see with one's own eyes all these hidden and inmost secrets unknown to current science and thereby receive a unique opportunity to rise to a height unattainable before.

It is enough to see the name Fermat in the title of our work to suppose something great in it. It was such a remarkable man that he could not create anything petty, even average: his mind shone with such a brilliance that he could not tolerate anything dark. It may be said that he is similar to the sun in a moment driving off the dusk and spilling the blinding light of its bright rays even into the abysm. Until now, everyone has been amazed by Diophantus and this is well deserved; but, no matter how great he was, it is a pygmy in comparison with such a giant who has come a long way around the world of mathematics traversed lands that have never been seen before. Vieta was praised by all those who *in our century devoted themselves to the study of algebraic operations, so for the glorification of some scientist it was enough to say that in the work on analysis he followed the thoughts of this author. But he also did not reach the heights of science which will become clear from the many examples explained below. Before Claude Gaspar Bachet I always bowed down as before a man of the subtlest mind; in addition, he was a close friend of mine and his research on Diophantus perfectly shows how astute he was in the science of numbers. But his gaze is weaker if you compare it with the lynx eyes of our Fermat, which penetrated into the most intimate depths.*

Jacques de Billy, 1670
Priest and Professor of Mathematics

Introduction

In the content of the book is presented the main theme consisting of about three tens items. This would be nothing special if all these items did not contain … the most real and incredibly loud sensations! But to say only this about this book would be to say nothing about it. Alone only illustration of the real (!) text in the margins of a missing book (see Pic. 5) we have restored, can cause a real shock among experts of the main theme! They might think: "Is this really the same book with Pierre Fermat's notes in the margins?". But no, this book is not yet available. And since we still managed to find out, what was actually written in its margins where Fermat's Last Theorem should be located, we depicted this recording by all means available to us. If we compare this restored text with the one that was published back in 1670 (see Pic. 3), then it becomes obvious that these are completely different recordings!

However, in our time, the Internet is also literally flooded with heart-rending screaming headlines about some sensations, which in fact are not, and their distributors resort to them only to raise the statistics of browsing. When it comes to science, if there are really sensations, then only in doses that cannot be captured by any statistics. The problem here is that the evaluations in the headlines are given by the distributors of information themselves who obviously should not be trusted. As for the content of this book, the situation here is principally different, since all the data here, assessments and conclusions can be checked by the most objective and incorruptible judge i.e. a regular calculator and anyone can always refer to it.

In particular, if there is a suspicion that the restored Fermat's record on the margins is nothing more than another fake among the sea of any other ones, they will prove to be not only nonconstructive, but also rejecting the opportunity itself to find out the real solution of the famous scientific problem. If this factor is not taken into account, then those who persist in such suspicions risk being in a very stupid position, since in this restored recording there is exactly what science still had no idea about. In fact, for science the FLT has always been just a puzzle, which for more than three centuries, could not be solved.

Such a scornful attribution of one of the fundamental scientific problems to the sphere of intellectual entertainment led to the fact that real science began to give way to ideas that have nothing to do with it. As a result, it turned out that all reference books and encyclopedias in unison and categorically tell us that the FLT problem has long been solved, but in fact science has no idea about how things really are. If this were indeed the case, the consequences would be so significant that they would radically change the state of all science in general as a whole!!!

Are you not believe? Well, judge for yourself, here is just one of these consequences. If the FLT is proven i.e. the solution in integers of the Fermat equation $a^n+b^n=c^n$ for $n>2$ is impossible, this equation turns out to be the only (!!!) exception from the more general case $A^x+B^y=C^z$ in which for any (!!!) given natural numbers x, y, z except of course $x=y=z>2$ may be calculate any number (!!!) of solutions in integers! And what now? Does science know, how to solve this general equation? Of course, no. Or perhaps science at least knows something about Fermat's equations for children with magic numbers? Or about the wonderful Fermat's binomial formula? Also no. However, the Soviet science fiction writer Alexander Kazantsev somehow incredibly way guessed about this formula, but mathematicians could not help him to derive it, so instead of a spectacular equation (see Pic. 1), he had to demonstrate an empty dummy.

Apparently, he did not even suspect that he had to ask for help not from mathematicians, but from children, then the result of his fantastic guesstimate would have appeared much earlier than this book where this formula is derived exactly in the appropriate place i.e. in the restored FLT proof from the Fermat itself! If this proof (obtained 365 years ago!!!), will learned by children studying in ordinary secondary school, they can easily cope with solutions of equations containing the magic numbers. These numbers, unlike some that mathematicians work with, are real because they obey to the Basic theorem of arithmetic (BTA). But the trouble is that current science does not even suspect that this most fundamental of all theorems has not been proven up to now!!!

But if science had become aware of this, then it would have no other choice as to accept BTA as an axiom since otherwise, science itself would simply disappear and then it could not be at all! Now, it will be a real surprise for science to find out that the problem of BTA proof was solved by the same Pierre Fermat and for this he used his own brand called the “descent method”. However, he could not divulge his proof since this would indicate an error of Euclid, in the proof of which he had it noticed, but this, not only at that time, as well as even now is inadmissible since gods by definition cannot be mistaken. It is also curious that without noticing the presence of BTA in the Euclid’s “Elements”, even such a giant of science as Karl Gauss exactly repeated the error of Euclid, what apparently also indicates his true divine origin.

In this book the proof of BTA obtained by Fermat is now like the FLT restored and the loopholes for penetrating into science of all sorts of pseudo numbers are closed, although it will not be easily to cleanse them because the precedent for them was created by none other than the greatest scientist and mathematician Leonard Euler! Indirectly in this was also involved Karl Gauss proving the “basic theorem of algebra”, which without these allegedly numbers called “imaginary” or “complex” would be wrong. Long before Euler and Gauss such well-known scientists as Leibniz and Cardano expressed their categorical rejection to this kind of "numbers". But they did not know that these Kazantsev’s non-existent beings disobey to BTA since only in 1847 Ernst Kummer told this very unpleasant news for the first time to the entire scientific world. However, for some reason this scientific world up to now stubbornly unwilling to get rid of the illusion of what really doesn't exist at all! For example, the Euler’s formula that causes delight $e^{i\pi}+1=0$ is in fact a complete nonsense that has nothing to do with science except perhaps to teach children not to believe in the reality of such tricks. Here even to them it is obvious that $e^{i\pi} = -1$ and this is certainly an obvious bullshit since the imaginary number $i = \sqrt{-1}$ being here makes imaginary and meaningless everything in where it is presented.

The main hero of our narration Pierre Fermat even in terrible dreams could not have imagined that only one of a whole hundred of his tasks [30] could even 325 years after the first publication of his works so much to discredit science, that it will turn out not only be incapacitated, but also literally standing in an head over heels position!!! Just in the period 1993-1995 it occurred immediately two events related to the FLT. The first is the Andrew Beal conjecture about the equation $A^x+B^y=C^z$, the proof of which allegedly allows to get FLT proof in one sentence. And the second is the Andrew Wiles’ FLT “proof” (which up to now nobody had understood), the news of which appeared in some incredible way in the newspaper "The New York Times" two years ahead of it! But then it was simply impossible to imagine what would happen when 25 years later it was suddenly found out that both of these events are pure misunderstandings!!!

Beal conjecture to the difficulty of its proof is suitable perhaps for school-age children. But this is just incomprehensible to the mind how it could not be proven up to now even for a prize of a whole million dollars!!! Another no less surprising side of this conjecture is the lack of understanding of how it is related to the proof of FLT, since what is written on this subject in Wikipedia is completely absurd. Nevertheless, Andrew Beal establishing such a large premium for his conjecture, clearly deserves universal respect, since with such a step he drew the attention of science on a theme, which had already taken place at Fermat in the above-mentioned restored FLT recording on Pic. 5.

The announced competition to prove the Beal conjecture does not allow us to clarify the solution of this problem in this book, because it can cause a real stir in the scientific world. Despite the simplicity of the proof of this conjecture, its consequences will be a loud sensation, since they will allow us really to get the simplest proof of the FLT. On the other hand, this will be too modest a result for the Beal conjecture, because its scientific potential is incomparably more powerful and impressive. To fix this situation to the best, this book will offer a more meaningful formulation of this problem, which called here the Beal Theorem, that not only confirms the correctness of conjecture, but also opens up the possibility of solving the equation $A^x+B^y=C^z$ for any natural powers except the case x=y=z>2.

As for the Wiles' FLT "proof", it rests only on the Gerhard Frey's idea, where again (for the umpteenth time in the past 350 years!) an elementary error was made!!! In this case, if something has been proven it is the complete inability of science to notice such errors, which must be teaching by schoolchildren. As a result, these events took place in such a way that on the FLT problem and its generalization in the form of the Beal conjecture, science once again became a victim of misunderstandings i.e. the current situation with the solution of the FLT problem is no better than the one that was 170 years ago, when the German mathematician Ernst Kummer provided proof of the FLT particular cases for prime numbers from the first hundred of the natural numbers.

With a such amount of knowledge available to current science, its helpless state seems as something irrational and even unthinkable. Nevertheless, it permeates whole of it through and far from only the FLT problem, but also in general wherever you poke, the same thing happens everywhere – science shows its inconsistency so often and in so many questions that they simply cannot be counted. The only difference is that some of them still find their solution, but with the FLT science has been stuck for centuries. However, the greatness of this problem lies in the fact that it, apart from purely methodological difficulties, points to some aspects of a fundamental nature, which have such a powerful potential that, if it succeeds in uncovering of it, science will be able to make an unprecedented breakthrough in its development.

Fermat paid attention to this aspect and was the first to notice even then, that science had no roots to support it as a whole. Simply put, the logical constructions used in solving specific problems do not have a solid support that determines the way, in which each separate branch of knowledge exists. If there is no such support, then science has no protection from the appearance of all kinds of ghosts taken as real entities. The Basic or as it is also called Fundamental Theorem of arithmetic is a vivid for it example. It would seem, what is simpler, one needs only to accept as an unchangeable rule that the numbers can be either natural ones or derived from them. Anything that does not obey this rule cannot be a number. Given that arithmetic is the only science that no other science can do without, it can be stated that all science cannot do without BTA at all! But science itself is not even aware of the fact that BTA is still not proven. And how do you think why? … This is because science simply does not know what is a number!!!

Even to people far from science, this obvious fact can make a shocking impression. Then the question obviously arises: if science does not know even this, then what can it generally know? In this book we'll explain what the difficulty is here and suggest a solution to this problem. This immediately draws the need for axioms and basic properties of numbers, which were also previously known, but in a very different understanding. After the definition the notion of number and axiomatics, proof of the BTA is required, since otherwise, most of the other theorems simply could not be proven.

As can be seen from this example, if a fundamental definition the concept of a number is given, then immediately a need appears to build an initial system defining the boundaries of knowledge, in which it can develop. It's like by musicians, if there is an initial melody, then the composer can create a complete work of any form and type from it, but if there is no such melody then there cannot be any music at all. In this sense, science is a very large lot of different melodies piled up into a one bunch, in which science itself is completely entangled and stuck.

But if science is built within the framework of the system laid down in it initially, then it will be as an unaffordable luxury a situation, when each individual task will be solved only by one method found specifically for it. The same problem took place in the days of Fermat, but for some reason besides him no one then bothered with it. Perhaps therefore, the tasks that he proposed looked so difficult, that it was not clear not only how to solve them, but even from which side to approach to them.

Take for example only one of Fermat's tasks, at the solution of which the great English mathematician John Wallis turned out properly to calculate the required numbers and even get praise from Fermat himself, any his task in that time nobody could solve. However, Wallis could not prove that the Euclidean method, applied by him, will be sufficient in all cases. A whole century later, Leonard Euler took up this problem, but he was also unable to bring it to the end. And only the next royal mathematician Joseph Lagrange had finally received the required proof. Even after all these titanic efforts of the great royal trinity, for some reason it remained unattended Fermat's letter, where he reported that the task is solved without any problems by the descent method, but how, nobody knows up to now!

In order to show how effective the descent method may be, in this book in addition to the proof of BTA, it was also restored proof by the same Fermat's method a theorem about the only solution of the equation $y^3 = x^2 + 2$ in integers, which could not be proven until the end XX century when André Weil has make it, but by another method and again of the same Fermat. If the problem proposed to Wallis had also been solved by descent method then the three greatest mathematicians, close to the Royal courts, would not have to work so hard. However, the result that they were able to achieve, may sink into oblivion due to excessive difficulties in understanding it and then all this gigantic work will slowly bypass the manuals as had already happened with the Cauchy proof of the Fermat's Golden theorem, about which it will also be told here.

There will also be touched upon a theme, which because of its seeming extreme difficulty, was as if ones did not notice and evade it. This theme about the special significance of arithmetic for the formation an abstract thinking, which obviously is of exceptional importance not only from the point of view of studying in the field of education, but also for understanding the essence of such a notion as mind. Having no such understanding, science as well as the story with imaginary numbers, is doomed to many failures. In particular, all attempts to create "artificial intelligence" of non-biological type will be in vain since it is impossible in principle! It will be shown in this book how Gottfried Leibnitz's truly ingenious conjecture, that thinking is an unconscious process of calculations, turned out to be true although only somewhat, because the mind cannot exist as a separate object or device and is a phenomenon of an ecumenical scale!!! If we now try to resume everything that we have mentioned here regarding arithmetic, then it will become clear, this is not only a science of sciences, but also a very effective sample for imitation.

Of course, in its present state it would be simply unthinkable, but taking into account what is stated in this book, such an imitation will become inevitable and a certain standard will be created, by which all sciences without exception will be built. It is not difficult to guess that the first point of this standard will be the definition the essence of given specific science. And of course, everyone will immediately think that it's very easily to find an answer to such a question at least by looking in some reference books or encyclopedias.

Aha, if it were so! Not to mention that the answers to this simple question for some reason turn out to be different (?), and to understand at least something from all them is hardly possible. Then it turns out that scientists specializing in some sciences simply do not know what they are doing? No, of course. They also like their predecessors use terminology, the meaning of which for some reason no one bothered to define and as a result of such a game without rules, sooner or later ghosts arise, which create the illusion of fantastic progress.

Well, and what about the sample for imitation? Considering the fact that in this book there is not even one, but whole two definitions of the essence the notion of a number, it is possible on this basis to formulate a brief definition the essence of arithmetic, say so: *arithmetic is the science about the origin of numbers and methods of computations*. Then from understanding the essence of numbers, one can construct their axiomatics and basic properties, which in turn will lead to BTA and other theorems arising from the needs for computations. In a similar way you can build also other knowledge beginning with basic notions and an essence of the science built on them.

Now for example, we need to use arithmetic as a sample for imitation in order to build, say, physics. To do this, we take as one of the basic definitions to this science as follows: *Physics is the science about the essence, properties and interaction of material objects*. Hmm … It seems here we stumbled upon an insurmountable wall because the definition the notion of matter does not exist. Philosophers spent a huge lot of paper, but all this without some use. However, as popular wisdom says, there is nothing to blame on others if they themselves have curved mugs. Physicists themselves can solve this problem without any special difficulties because no one else will do it for them.

They simply accept as an axiom that *all consisting from matter has such properties as mass and energy* and so simple the whole problem will be solved. Well, and what about the definition the essence of these properties themselves? But this is still Sir Isaac Newton has very well worked and even used the style of presentation along with approaches from Euclid himself! And now, standing on their shoulders, it's not at all difficult for us to reveal the essence of these notions especially after physicists have the problem with the units of measurement solved. Indeed, in arithmetic it is only implied that all calculations must be carried out in the appropriate units of measurement while in other sciences these units must always be concrete.

For example, in informatics Bit is used as the unit of measure, but here scientists also screwed up. Since the times of Claude Shannon, it is considered that the quantity of information is measured by Bits, but given that the notion of information is not defined at all, it turns out that they measure unknown what. However, in fact, it is all very well known to everyone that by Bits is measured the capacity of memory of information carrier. But how to measure the quantity of information itself is a problem, the solution of which will largely determine the possibility of implementing the most powerful technological breakthrough in the entire history of our civilization!!!

A term "technological breakthrough" is from the field of economics, but this science is only a ghost if only because it uses as units of measurement only meaningless titles. Economic crises in contrast to the devastating storms, hurricanes and tornadoes, have no natural origin since they are the consequences of people activity who do not understand what they are doing and therefore are not able to prevent them. This book will offer a way to solve these problems from the point of view of the possibilities of building not sham ones, as they are now, but real informatics and economics built in the image and likeness of arithmetic.

From that we have already said, many people will probably think that all this looks like something too fantastic to be a reality. But everyone so thought also about Fermat. When he offered his task to someone, that someone discourses very simply: well, if Fermat is a Gascon, it means a prankster. In Simon Singh's book about the FLT, Descartes allegedly called Fermat a boast man, what confirms this common opinion, but his exact phrase was: "… unlike Monsieur Fermat, I'm not a Gascon". If this introduction of ours also will causes distrust or will be perceived as humor then this is exactly what is needed, because it consistent with the spirit of our main hero.

On the other hand, all the themes touched here, are too fundamental to be disclosed in the traditional style of scientific monographs. Then it would have turned out something like, say, the British Encyclopedia or the complete works of Leonard Euler consisting of about 800 volumes, which for more than 200 years anyone had not been able to publish completely at least once. So that our works would not be lost at all, we had to take an extraordinary step i.e. to use for this book an unusual literary genre called here *the scientific blockbuster – a combination of narrative in the sharp style of artistic prose along with the separate fragments of purely scientific content*.

How one would not to relate to this kind of innovation, here is the result already evident: main themes of the book's content are presented in detail in 6 points of the "Resume" section and 100 points in the list of Appendix V, which is made up of what will be clearly new to current science. In addition, in order to densify the main content, 172 comments were carried out and three separate miniatures were added as applications, which usually have a reference character but here, they are presented as a natural continuation of the main part of the book, without which it would be incomplete.

The plot of the first miniature is very interesting because in the proof of BTA from German professor Ernst Zermelo (a student of Max Planck himself!) 1912 there is such a barely noticeable error that upon learning of this the authors of the textbooks will be extremely surprised. But no less surprising here is the fact that this error in fact is the same as in the Gerhard Frey's idea for the FLT "proof" by Andrew Wiles 1995 only more veiled. Thus, the mistake coming from 1912 and appear in 1993 turned out with just terrible consequences, which completely destroyed the "solutions" of two fundamental problems that the scientific world so carelessly allowed himself to admit.

The second miniature is no less curious, because it describes in detail two proofs of the same particular case of FLT for n=4, first by Leonard Euler and then by Pierre Fermat in the reconstruction of I. Bashmakova. Both proof as twin brothers are built on the Pythagoreans identity and in both the descent method is used. They differ only in the intricacies the logic of output to the same end result. These intricacies, although different, are quite complex, what indicates the highest skill of their authors. But the end of this miniature is simply amazing. And indeed, this proof can be obtained from the same Pythagoreans identity literally in a one line (!!!), and this very line is just in the FLT recording we restored in the margins of the book shown in Pic. 5.

In addition to the Euler proof of the special case of FLT presented here, it is also added the full text of all Euler's proofs related to Fermat's grandiose discovery of the truly magnificent properties of primes 4n+1 type. This work required the utmost exertion of all Euler's creative and physical powers within seven years, but the most important proof that these numbers always consist of the sum of two unique squares, is presented by him in such a way that it is unlikely that anyone except himself understands its essence. From Euler's letter to Goldbach with this proof, at first no one understood anything at all, and after the corrected version received by Goldbach in another letter, all the experts tacitly accepted his proof, although it is far from obvious and besides, numbers of this type should be the sum of two *unique* squares, but about it there is not a single word at all.

Finally, the third miniature is a journey into the past. There will be a lot of surprising and even shocking things, but here we will pay attention to only one a moment. This is the Fermat's proof of his most grandiose discovery in the field of prime numbers, which is unknown until now and here it is presented by a special way and in amazingly beautiful form. The story about this through the mouth of Fermat's son Clement Samuel with a cherry on the cake in the form of a spectacular equation will make just as colorful an impression as the beauty of nature.

The method of vestment in the verbal form of the content of this book, chosen by us, although it requires an immense strain of all forces from the author, still yields a result, in which a small volume of the book carries the knowledge of thousands of scientific monographs! Perhaps such a precedent will be the first and the last, and in this sense, it is not a competitor to traditional scientific monographs. However, in essence it is just following the simple advice of a classic on choosing a style of exposition, where for words it would be cramped and for thoughts spacious.

The usual technical language does not achieve such an effect and this requires a higher level of literature accessible only to the elect, for example, such as Alexandre Dumas the Father. In one of his books Dumas even argued that writers understand history better than historians. Wherein, he has fib so famously and godlessly that historians could only smirk. However, in fact Dumas turned out be right because the lion's share of the history set forth in the thick books did not really have place, but was simply invented and this fact also found a place in this book.

One of the features of our literary creativity is the mandatory presence in it of riddles, which are announced, but not disclosed. There is a whole 15 of such riddles in this book, and they are marked (*) in Appendix V (points 18, 19, 26, 27, 39, 41, 42, 43, 44, 45, 46, 47, 48, 69, 74). Questions and problems are focused only on key points that are of principled importance. When such a moment of truth comes, it may give the impression or even bewilderment that science has not noticed such simple things for centuries. But this is where the power of real science lies since the Almighty in his creations always follows only paths with the shortest and simplest solutions.

In real life false knowledge often takes the place of real ones. Behind this lies a lot of danger and unnecessary problems. However, they will bypass those who will be able to understand this book. However, if it will not work for someone, it's best to turn to children for help and then they will simply be amazed at childish abilities to penetrate into corners of the unknown, which are completely inaccessible to most people. But children themselves do not realize that at their age all people are wizards for whom there are no insurmountable obstacles. Think of although a three-year-old Gauss who made accurate calculations for his father a pipeline engineer. And others can do it too, but they just don't know about it!

In this book many different names are called, which created history by the will of Providence or case, and just because they turn special attention to themselves they deserve every respect, no matter in what circumstances and how they showed themselves because otherwise, there would simply not be events, from which the plot of our narrative was formed.

From what we have already talked about science here, it will look completely unattractive. Moreover, it will be presented as the source of all troubles, and sadly this is the harsh truth. But if the question about the place of science in society is not raised and, in any way, not clarified, then a catastrophe allegedly coming from scientists, will become inevitable and the very existence of our wonderful world will lose all meaning. This is not at all some formidable warning or apocalyptic prophecy, but merely an ascertaining that science is the only (!!!) field of human activity that predetermines all their other varieties!

Thus, in an intelligent society the highest priority of science must be ensured and supported by all available means, otherwise, it will receive only a global confrontation of ignoramuses balancing on the precarious verge of mutual destruction. And what we have now? Only that the management of society goes not in accordance with the objective laws of the world, but through blatant incompetence, irresponsibility, bribery, adventurism etc. Where is here the science? It is not even near visible anywhere. If even the applied science, which has been robbed by money-lenders to the last thread, somehow can still cling to its existence then for a fundamental science a long time ago there are no any prospects at all.

But perhaps scientists need to offer something themselves so that the fruits of their labor will be appreciated? Ha-ha-ha! There is a well-known case when Gregory Perelman had published without any conditions in free access the proof of the Poincare conjecture, which more than a hundred years apart from him could not be obtained by scientists. However, instead of (already offered to him!) a premium of US$ 1 million he got nothing. The press reported that allegedly he had himself refused under a fictitious pretext, but for some reason all thought that he was just an eccentric. However, in fact he did not even think about refusing, apparently naively believing that the prize he has fully deserved.

However, he did not take into account that in a society, in which the leading positions have not scientists, but usurers and bribe-takers, scientific discoveries that do not give immediate return with money, are even for free nobody need. In fact, they really offer prizes not for scientific discoveries, but for a well-known name that can be exploited in their own interests. Yet Perelman in this story brought the initiators of the award to a clean water after he offered to share prize with another scientist related to his scientific discovery and then it became obvious that in fact the refusal did not go at all from him, but from imaginary benefactors.

In terms of determining the value of scientific discoveries, there simply cannot be any illusions that nobody really needs them. In the obviously dying world of usury, theft, gain, speculation etc., the attitude to science can be only as it is. There is no doubt that the premium for proof of the Andrew

Beal conjecture will also not be paid for its intended purpose. Are you don't believe? Well, it's very easily to check!

In this book there are examples of such calculations, which leave no room for doubts that it would be impossible to carry them out without knowing the essence … no, not of a conjecture, but of a much stronger statement called here “The Beal Theorem”! If the aim of the Beal Prize is really to get this impressive scientific discovery, then the organizing Committee in the face of "American Mathematical Society" would be easier do not rely on the propitiousness of mathematical editions, and just to request it directly from the author of this book.

This way would be clearly simpler and better since the proof of the Beal conjecture is too elementary and not so significant for science as the proof of the Beal Theorem, which would be much more useful, productive and impressive with the same end result that is required in conditions of the Beal Prize. The risk of arising another fake in this case will be excluded, but if nothing to be done to solve this problem, the initiator of the prize Mr. Andrew Beal may never wait to achieve his goal. Besides, it should be borne in mind that expert evaluation of the Beal conjecture proof does not require such obviously excessive precautions, because this task is for children from secondary school. What is written in this book is more than enough to make sure that this task has not any difficulties for the author.

It is very curious in this sense, how science will react to the appearance here of the FLT proof, performed by Fermat himself! And this is in conditions when as many as 18 (!!!) the most prestigious awards for obviously erroneous proof 1995 have already been presented! Of course, no one is immune from errors and we will show here how such pillars of science as Euclid and Gauss made the most elementary blunders in proving the Basic theorem of arithmetic, as well as Euler, who blessed the use in algebra of “complex numbers” , which are not numbers due to the fact that they do not obey to this same Basic theorem. However, Euler wasn’t aware of it yet, but his followers know this perfectly well for the two hundred years, nevertheless no one even had a finger stir to correct this mistake.

As for the not needed scientific discoveries, many people simply do not know that they can live quietly and consume all the vital resources they need only until the knowledge resource, accumulated in society for a given level of its development, will be exhausted. And after that, in order to keep what has been achieved, the stronger countries will attack the weaker ones and live at the expense of their plunder. But this would not have been necessary at all if these “strong” countries had enough knowledge. Then they would not have conflict with the rest of the world since all the necessary resources would be provided in abundance by science.

On this we will complete our introduction, but we will give it such a secret impulse that will allow us to perform a real wonder! … no, even two! We can call these wonders here by their proper names because our eternal opponents from the complete lack of real science by them, are simply incapable of this.

As a result, they will learn about the realization of the most grandiose technological breakthrough in Russia in the entire history of our civilization, with unlimited potential of development effectiveness for the immense future. The notorious “valleys”, “techno parks”, “incubators” and the like ghosts for such breakthroughs are unsuitable in principle. But still earlier, another wonder will happen when Russia literally in a couple of months, on the wreckage of collapsing today the world usury financial system, will create a new one, in which no any international money will be needed and all countries in international trade will use only their national currencies.

Are you again don't believe? Well, you can see for yourself because the book is in your hands!

1. The Greatest Phenomenon of Science

Usually, the science's image is represented as an ordered system of knowledge about everything that can be observed in the world around us. However, this image is illusory and in fact there is not any orderliness in science since it is formed not by the development of knowledge from the simple to the complex, but only by the historical process of the emergence of new theories. The classic example is the Descartes – Fermat analytic geometry, where compared with Euclidean geometry, science sees only an analytic-friendly representation of numerical functions in a coordinate system, but does not evaluate the qualitative transition from naturalized elements (point, line, surface, etc.) to numbers.[1]

It would seem that this is so insignificant that it cannot have any consequences, but ironically, it was after the expansion of the numerical axis to the numerical plane, when science was hopelessly compromised, because it suddenly became clear that such a representation of numbers does not obey to the Basic theorem of arithmetic that the decomposing of an integer into prime factors is always unique. But then a corresponding conclusion should be made that no any numerical plane exists and everything connected with it should be written off to the archive of history.

But it's really impossible! If there is no orderliness in science, then there is no reason to link new knowledge to earlier ones. Therefore, it is not at all news to the world of scientists that for the numerical plane the Basic theorem of arithmetic is not acted. This was known a century and a half ago and it never even occurred to anyone to abandon this idea. During this time, so much has been done that it's so easily to take it all and throw away is in no way possible because many "experts" with their "scientific" research can lose their jobs and all monographs, reference books and textbooks on this theme will at once turn into tons of waste paper.[2]

Yes, not one of the scientists can be surprised by the fact that the Basic theorem of arithmetic is not acted, because they have already accustomed not only to such things. But they will be very surprised, when they know that nobody can prove BTA so far! All the "proofs" of this theorem in textbooks and on the Internet are either clearly erroneous or not convincing. But then it turns out that on the one hand, science deprives itself legitimacy since it does not recognize the Basic theorem, on which it itself holds, but on the other hand, it throughout all its history simply was not aware of the fact that it has no proof of this theorem.[3]

And what now to do? Can this blatant fact be perceived otherwise as the degradation of science in its very foundations? To some people such a conclusion may seem too categorical, but unfortunately for current science, this is even very mildly said. What a marvel, some theorem doesn't act? And what about when the law of conservation of energy doesn't act? Current astrophysics simply does not present itself without the "big bang theory", according to which all the galaxies in the Universe are

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[1] Naturalized geometric elements form either straight line segments of a certain length or geometric figures composed of them. To make of them figures with curvilinear contours (cone, ellipsoid, paraboloid, hyperboloid) is problematic, therefore it is necessary to switch to the representation of geometric figures by equations. To do this, they need to be placed in the coordinate system. Then the need for naturalized elements disappears and they are completely replaced by numbers for example, the equation of a straight line on the plane looks as $y=ax+b$, and the circle $x^2+y^2=r^2$, where x, y are variables, a, b are constants offset and slope straight line, r is the radius of the circle. Descartes and independently of him Fermat had developed the fundamentals of such (analytical) geometry, but Fermat went further proposing even more advanced methods for analyzing curves that formed the basis of the Leibniz – Newton differential and integral calculus.

[2] Under conditions when the general state of science is not controlled in any way, naturally, the process of its littering and decomposition is going on. The quality of education is also uncontrollable since both parties are interested in this, the students who pay for it and the teachers who earn on it. All this comes out when the situation in society becomes conflict due to poor management of public institutions and it can only be "rectified" by wars and the destruction of the foundations of an intelligence civilization.

[3] The name itself "the Basic theorem of arithmetic", which not without reason, is also called the Fundamental theorem, would seem a must to attract special attention to it. However, this can be so only in real science, but in that, which we have, the situation is like in the Andersen tale when out of a large crowd of people surrounding the king, there is only one and that is a child who noticed that the king is naked!

flowing away like fuzz. And such a crazy phantasmagoria is quite seriously presented today as one of the greatest "scientific" achievements, and fig leaves like "hidden energy" and "dark matter" easily cover the problems with the notorious conservation laws.

Against the background of the truly outstanding achievements of science there is no doubt that this virus of dark misfortune, which penetrated into its very foundations, could not have emerged from nothing and was clearly introduced from the outside. The malicious nature of the virus is disclosed by the fact that it always hides under the guise of "good intentions." And if that is so, then the task of getting rid of the misfortune is simplified because these are just the intrigues of the unholy, from which the real science always had sufficient reliable immunity.

But for this particular virus this immunity began to act in a very special way. Suddenly out of nowhere, there appeared a simple-looking task called “The Fermat’s Last Theorem” (FLT), which no one could prove despite the promised bonuses and honors. It simply scoffed at everyone who tried to find a solution regardless of whether it was an ambitious candidate for the prize or the greatest scientist. With the FLT many scientists were even afraid to deal in order not inadvertently to tarnish their reputation.

This fascinating game with a knowingly failure result dragged on centuries and in the end, everyone was so tormented that it was necessary somehow this problem try to close. Very serious people made a decision – the problem is to be solved and bonuses are to be paid. No sooner said than done. However, what happened next will be told in the next part of our work. But it will be only a preamble because in order to penetrate the essence of this amazing phenomenon we will have to come back in the past in some unusual way. And then as a result of our research, it will turns out that this task was solved long ago in the 17th century when Louis XIV the king sun began to rule in France and two Gascons faithfully served him, one of them is the well-known from novel A. Dumas is the royal musketeer Monsieur D'Artagnan and the other is his same age and countryman Senator from Toulouse Monsieur de Fermat.

The history did not preserve for us in writing everything that would be especially interesting to us, therefore, nothing remains, but to try to restore some events at that in a very unusual way what about we will also more tell. However, it is well known that this senator during his lifetime became famous for offering simple-looking arithmetic tasks to noble grandees, which for some reason no one could solve. But apparently, he didn't had time (or even perhaps he didn't want) to tell anyone about that wonderful and non-proven until now theorem therefore it is also often called the “the Fermat’s Last theorem”.

Especially curious is the fact that not a single piece of paper has been preserved from the manuscripts of his scientific works on arithmetic and even those that were published after his death. The only exceptions were letters collected from different respondents. This strange fact indicates that some amazing and even incredible course of events took place, which led to such a situation and the establishment of only this fact alone significantly changes the whole picture, which presented to researchers so far.

They even believed that Fermat could not have a proof of his Last theorem and justified it with all sorts of arguments. But then they needed to be consistent and insist that Fermat also could not solve all other his tasks since for his justification he has not left us any explanation. But if they were solved by such giants of science as, say, Euler or Gauss, well, then it is quite another thing and we could assume that Fermat also has solved them. But if even they failed, then science in no way cannot afford to trust words that look like bluster.

In our research we will go the other way and we will proceed from the fact that the proof of Fermat’s Last theorem, without any doubt, should have been written down on paper at least in a sketch version. But if this is so, then where could it have disappeared moreover along with all the other papers? The answer to this question can shed light on the healing of the above-mentioned misfortune, which led to the fact that for unknown reasons this very proof for as much as three and a half centuries has become not only an unsolvable problem, but also a real stumbling block for science.

The riddles that we now have to explore seem at first as an accidental collision of all kinds of large and small stories, but these seemingly intricate events have their own rather rigid logic. It so happened that Fermat's life and activities coincided with a turning point in history when a slow and very painful transition to the Renaissance took place after a long period of terrifying oppression by the Inquisition, which did not tolerate advanced scientific thought and have organized in France mass destruction of Protestant-Huguenots by Catholics.

Taking into account this circumstance, it is possible to explain such facts and events that from the point of view of a later time look as very strange and not able to understand. In particular, it should be noted that in those times, especially for people of ignoble origin, it would be very dangerous to have at home even completely harmless notes with formulas and calculations that could be interpreted as a very dangerous for their owners' recordings of heretical content.

Pierre's Father Dominique Fermat was a wealthy merchant, but did not have a noble title. In 1601 his son Pierre was born, about which there is an entry in the church book, but his mother Françoise Cazeneuve and her child died not having lived after giving birth to three years. If the child had survived, then without a noble origin, he would have no chance of becoming a senator let alone a great scholar. And when after the loss of his first wife, Dominique married Claire de Long having noble roots, then this ensured a very opportunity that the future celebrity would appear [16].

Pierre Simon de Fermat was born not in 1601 as it was believed until now, but in 1607 (or in 1608) [1] in the little town of Beaumont-de-Lomagne near Toulouse. From childhood he stood out for such talent that Dominique Fermat did not spare the funds for his education and sent him to study first in Toulouse (1620 – 1625) and then in Bordeaux and Orleans (1625–1631). Pierre did not only study well, but also showed brilliant abilities that together with his mother's kinship and financial support from his father, gave him every opportunity to get a best education as a lawyer.

During his studies the young future Senator Pierre Fermat was very keen on reading scientific literature and was so inspired by the ideas of great thinkers that he also himself felt a desire for scientific creativity. In order to learn more about what particularly interested him, he had mastered five languages[4] and read with enthusiasm the works from the classics of that time. As a result, he deservedly received the highest education that just was possible in those times and deep down he cherished the dream of being able to continue work in the field of science.

If the support of Pierre Fermat's career had ended on that, then there could be no question of a future senator since in those times even simple lawyer activity demanded the highest royal deigning. From this it becomes clear why the decisive step in Pierre's parental care was his marriage in 1631 to Louise de Long, who was a distant relative (the fourth cousin) of his mother. It is clear that such a decision could not be spontaneous especially since such kindred marriages could be concluded only with the permission of the Pope of Rome. And once again the Dominic Fermat's money solved this not simple problem.

Louise's father was an adviser to the Toulouse Parliament and being in the service of King Louis XIII, received a noble title, so Pierre had no problems with employment. But it would be a delusion to expect that also further everything will go on easily and smoothly. After the end of the study, marriage and the beginning of work, the reality seemed to Pierre as at all not so rosy. The gray days of the hustle and bustle of earning money for daily bread went day after day and did not leave any hopes to be engaged in science. And then it was still a very great good to have within the framework of lawyer activity the ability to support though not a luxurious, but still a well-off life in those difficult times for France.

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[4] On a preserved tombstone from the Fermat's burial is written: "qui literarum politiforum plerumque linguarum" – skilled expert in many languages (see Pic. 93-94 in Appendix VI).

A new danger for Pierre appeared unexpectedly. The next plague epidemic claimed the life of his father-in-law and this could have a very bad effect on his fate. However, by that time he had already managed to establish friendly relations with other senators what opened for him the way to parliament and as a result it made possible to turn the misfortune in his favor. With the help of a fair amount of money, he still managed to take the vacant position of an official in charge of receiving complaints in the cassation chamber of the Toulouse parliament.

The biographers of Pierre Fermat rate his career as simply brilliant, but at that they lose sight of one very significant detail. Exactly such a career tightly closes him all even the slightest opportunities to be engaged in science. They did not take into account the fact that there is a royal directive forbidding the posts of councilors of parliament for the people engaged in scientific research that may contradict the Holy Scriptures. But since Pierre became a senator, this will put a big fat cross on his dreams of being engaged in science on a professional basis. He will carry this cross for the rest of his life.

Moreover, as a Catholic he should not commit any mortal sin and is obliged to confess regularly once a year about the pardonable sins committed by him. As such a pardonable sin Pierre reports at confession about his moderate idleness after reading the books by Diophantus of Alexandria “Arithmetic” and “Tasks undertraining and pleasant, related to numbers”.

Pic. 6. Diophantus of Alexandria

The risk of falling into disfavor by such a sin fall was small because the book was published by Claude Gaspard Bachet de Méziriac a flawless in every respect a high-ranking linguist and future member of the French Academy established by Cardinal Richelieu in 1635. Here of course, there will be a question about the secret of confession. But if even in our time with respect to the Catholic Church this question looks very naïve, then what is to say about the times when the supreme executors of the royal power were cardinals. All priests were obliged to inform the authorities about what their parishioners live and especially officials in government posts. Information from the priests was also controlled, for which authorized inspectors were sent to the places.

Pic. 7. Bachet de Méziriac

It is understandable that Pierre could not expect anything good from meeting with such an inspector, but he had no choice and was ready to put up the complete impossibility of his dream. But then of course, he could not have known that he was destined to another fate and it was to decide at that very moment. It is even difficult to imagine his amazement when an arrived inspector turned out to be the priest Marin Mersenne … a passionate lover and connoisseur of mathematics!!!

Pic. 8. Marin Mersenne

Pierre took it as the supreme wonder bestowed on him from heaven by the Almighty. And how else could this be understood since Reverend Father Mersenne managed miraculously to organize for him the possibility of correspondence with René Descartes himself as well as with other elite representatives of the French creative aristocracy what about he could not ever to dream. Pierre went through the test brilliantly when he was able to solve several problems at the request of Mersenne and in particular quickly calculate some of the so-called perfect numbers moreover, also those that were previously unknown. Hardly anyone else could to solve or at least somehow cope this task.

Historians in their studies see only pure randomness in the coincidence of interest to the numbers of Mersenne and Fermat, and Mersenne himself in their presenting is a weirdo acting on his whim. However, in real history so does not happen and there should be a more reasonable explanation of events. In this sense, it would be much more logical to believe that Mersenne was no more than a performer of some instruction from above, and since he came from church nobility, only one person could give such an instruction to him – it was no one other as Cardinal Armand-Jean du Plessis Duc de Richelieu!

Pic. 9. René Descartes

This implies the activity of the association of learned nobility created by Mersenne, could not be just his initiative, but was sanctioned by the highest authorities of that time, otherwise this activity could not be deployed or it would curtail after the death of Mersenne in 1648. However, his brainchild continued to function for a long time and successfully until the creation of the French Academy of Sciences in 1666.

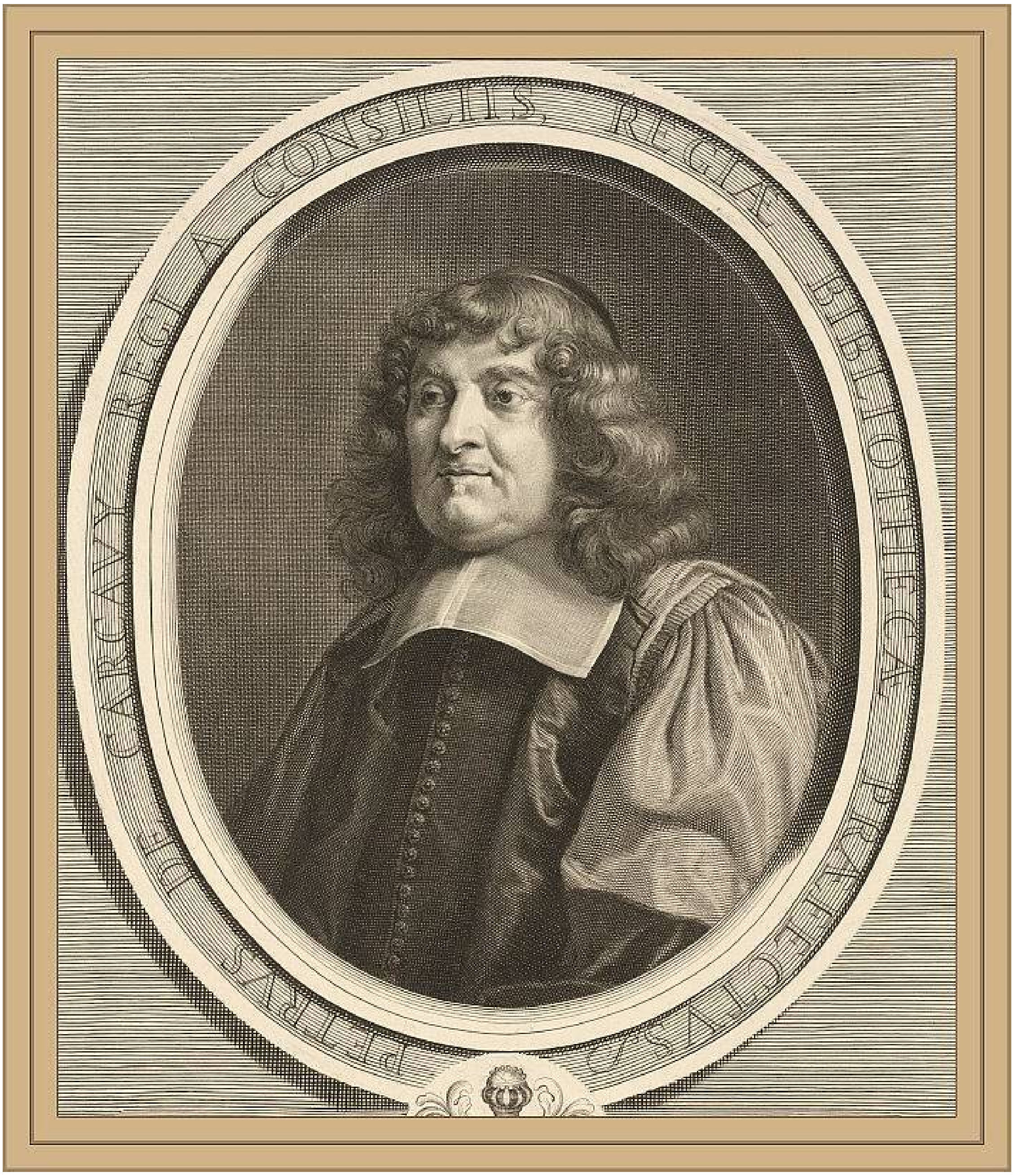
As for Pierre Fermat who became a Senator, he found himself in a difficult position. His abilities were now in demand, but he could develop them only at his own expense and without the right of publication because no one has repealed the royal prescription for restricting appointments to the posts of advisers to parliaments and he had no other means to earn a living. So, for his future opponents, he will appear as a recluse who does not want to share the secrets of his scientific discoveries. Even his friend Blaise Pascal in one of his letters sincerely wondered why he did not publish his works. To this Fermat also sincerely replied, he did not at all want his name to appear in print. Well, he really could not refer to the royal directive, which does not allow any scientific activity on the position he occupies.

Pic. 10. Blaise Pascal

For Fermat everything was happened so that he had no opportunity to solve this problem otherwise as by his direct participation in the preparation of the royal decree on the creation of the French Academy of Sciences. This is indicated by his correspondence with Mersenne and Pierre de Carcavy who was involved in the preparation of this decree. Fermat received a desired noble title only after 17 years of diligent service becoming in 1648 a member of Edicts House, which met regularly in the little town of Castres near Toulouse. But this promotion only increased his workload and further limited his opportunity for science activity.

But paradoxically in this life drama is distinctly seen a truly divine providence having lay a special mission to Senator Pierre de Fermat aimed at saving science from destruction. At that early age the science was still seemed as a beautiful tree, which by growing became more and more valuable and attractive. But with the development of science the features of perfection and harmony inherent in it, began to fade and the image of the beautiful creation of the mind more and more resembled a helpless little freak.

Pic. 11. Pierre de Carcavy



These first signs of trouble were noticed else by Fermat since controversies in his correspondence with colleagues appeared almost on empty place. It became clear that this tree has almost no roots. This means that science does not have a sufficiently strong foundation and for it there is a threat of the fate of the Pisa Tower. Then, in order for this magnificent building of science to serve its intended purpose, all creative forces will have to be used not for development, but for preventing its complete collapse.

For Fermat this theme was going past the limits of his physical possibilities and he considered it only from the point of view of generalizing methods for solving various arithmetic problems. It is so, because arithmetic is not some separate science, but the basis for all other sciences. If we have no arithmetic, then we have no any science generally. In this sense, the arithmetic tasks proposed by Fermat are of peculiar importance. Their peculiarity is that they teach people to think in general categories i.e. to find methods regulating the possibilities of computations for solving a wide range of tasks.

And here is an amazing paradox. About Diophantus who gave solutions to nearly two hundred completely not simple arithmetic tasks, now, if anyone remembers of him, then only in connection with the name of Fermat. But about Fermat himself, who did not leave any single (!!!) proof of his theorems,[5] all and sundry are constantly discussing for the fourth century in a row! Very few of those who were able to solve although one Fermat's tasks, secured for themselves world-wide fame, but countless number of people who suffered fiasco, cannot find for this any rational explanation and they have no other choice, but only simply to ignore this very fact.

But how could such an amazing phenomenon appear in the history of science when a man, who was not even a professional scientist, became so famous? To see here only an accidental combination of circumstances, would be clearly unwise. It is much more logical to proceed from the fact that at some stage in his life, Fermat began to realize that if his plans for publishing his research were carried out, the fate of Diophantus, which was already then almost forgotten, awaits him at best. If about Fermat anyone will also remember, then only against the background of derogatory and even caricature opinions of the "experts".

In fact, it is all happened just so, but the effect was the opposite. No one could have imagined that thanks to Fermat a fascination with mathematics would take on such a mass character. The more his opponents sought to belittle him, the more popular his name became. Even the feats of D'Artagnan, which were fictional by A. Dumas, were simply childish pranks compared to what his fellow countryman Toulousean Senator Pierre de Fermat did in reality. And yet, how could this provincial judicial official be able to achieve such an amazing result?

It is very simple since he was a lawyer, he did everything exclusively and only legally, therefore he has left to himself all the works, in which his opponents could see recordings of "heretical content". In addition, he was not only an outstanding mind with a lot of life experience, but also a Gascon. And it is well known that people of this type even the very serious doings can present in such an unpretentious and humorous wrapper: Yes, sometimes I'm reading Diophantus' "Arithmetic" at leisure and made notes with some ideas following the example of the esteemed and Right Honorable Claude Bachet who performed not only the Latin translation during the preparation of this book in 1621, but also added his own remarks.

Fermat did exactly the same i.e. had prepared for publication, as if were not his own works, but the same "Arithmetic" of Diophantus (see Pic. 96 in Appendix VI) with the same remarks of Bachet and has added to them the 48 his remarks. Everything was prepared so that any claims to this book or to him the Honorable Senator Pierre de Fermat simply could not be. But when the book was published, then unlike its previous editions, it stirred up the entire scientific world! Those comments made allegedly in passing on the margins of Diophantus' book, turned out to be so valuable that they allowed scientists to develop science very noticeably using Fermat's new ideas for hundreds of years! And everything would be just perfect if it were not this his Last Theorem not amenable to any comprehension in scientific circles.

It would seem, what might be unusual here? Such unresolved problems in science are simply cannot be counted. But the fact of the matter is that the author of the theorem himself announced that he had the "truly amazing proof", but science cannot get although any for 350 years!!! It is only in the mass consciousness the author of the theorem is a real triumphant, but for science it's like a bone in the throat. Here are already present obvious signs of illness. What kind of science is this, which for hundreds of years cannot to solve the school task? It would be OK if only one this task, but science cannot also recognize the obvious fact that it does not have the basic knowledge necessary for this, which Fermat discovered yet in those distant times.

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[5] It is believed that Fermat left only one proof [36], but this is not entirely true since in reality it is just a verbal description of the descent method for a specific problem (see Appendix II).

Science lost not only the ability to comprehend, but also to orient in the events around it. How is it so that we have no knowledge, there are a whole mountain of them! This is for sure, “knowledge” was accumulated so many that to understand and assimilate all this wealth has become beyond human strength and capabilities. But in fact, everything is just the opposite. There is a very noticeable lack of real knowledge and the most part of all what has been accumulated, is empty grinding of many problems, which either have no solutions at all or else worse when dubious ideas are taken as the initial ones, on which mind-blowing theories are built, what naturally generating all sorts of paradoxes and contradictions. Then scientists are trying with all their might to overcome them, but for some reason if something work**s** out for them that only with the help of even more mind-blowing theories.

Such an unusual character of our perceptions concerning to science, can cause a very negative reaction. But here we can confess that we had very good reasons for this because we managed to look in those very “heretical recordings” of Fermat. For greater persuasiveness we directly here will show one of the examples of our capabilities and accurately reproduce the real text of the most intriguing recording of the Fermat's Last theorem in the margins of Diophantus' "Arithmetic", which instance did belong to the author and disappeared unknown whither. So, in this place (see Pic. 5), we gain sight of several notes to the task under the number VIII made in Latin at different times. In translation they look like this:

1st entry: *However, it is impossible to decompose C into two other C or QQ into two other QQ. Both proof by the descent method.*

2nd entry: *The second case is impossible because the number 2aabb is not a square.*

3rd entry: *New solution to the Pythagorean equation AB=2Q.*

4th record: *It may be computed as many numbers aa+bb–cc=a+b–c as you like.*

5th entry: *And in general, it is impossible to decompose any power greater than 2 into two powers with the same index. Proof by a key formula method.*

6th entry: *However, you can calculate as many numbers C+QQ=CQ as you like.*

Now this restored text in margins of book can be compared with the text published in the edition "Arithmetic" by Diophantus with Fermat’s comments in 1670 (see Pic. 3 and at the end of Pt. 4.2):

However, it is impossible to decompose a cube into two cubes or a biquadrate into two biquadrates and generally any power greater than two, into two powers with the same index. I have discovered truly amazing proof of this, but these margins are too narrow to put it here.

But then it turns out the recovered text is not at all the same that was published. Well, of course not that one! It's clear, if you publish the real text of the remarks made in the margins of the book, then no one will understand anything because that who writes them, does it not for someone, but only for himself. On the other hand, it is obvious that the content of the recordings in the margins is so that they could not be made in the course of reading the book and are the result of a very voluminous and many years of work that was done separately. It is obvious that in addition to these short notes there is yet a whole bunch of papers in draft and finishing versions with brief or detailed explanations. These papers have not always been prepared for printing and they still need to be brought to the desired state. Hence it is clear why the text was edited accordingly for publication in 1670. From the real notes all was removed that reveals the method of proof and the sequence of solving individual tasks, which have eventually led to the discovery of the FLT.

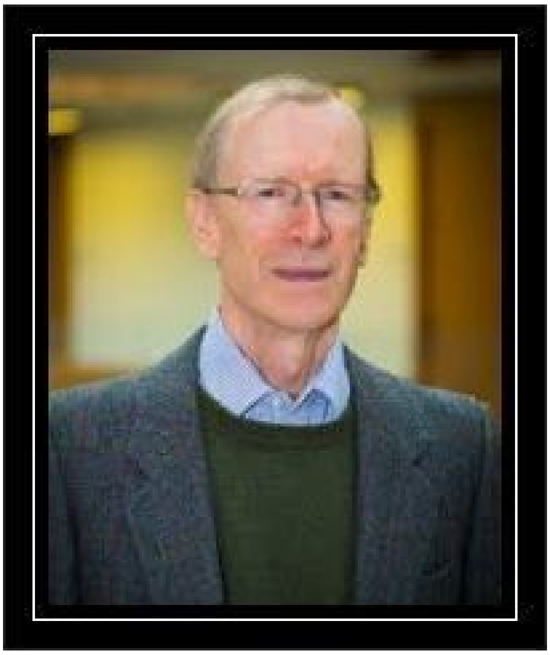
The restored remarks follow in chronological order and may diverge in time over years. The margins' records of the book were made after they were prepared separately, but it was not intended that they be published in the same view. On the contrary, in the final formulation of the FLT everything that could be concealed from the history and components of this brilliant scientific discovery, was completely removed. Only the final result has been remained, which turned out to be beyond the powers of all subsequent science right up to the beginning of the XXI century!

If this reconstruction of the original FLT recording on the margins of the book appeared 30 years earlier, it would have caused a quite stir in the scientific world since the sixth entry develops (!!!) this theorem to the general case with the different of power's indexes! However, this stir did nevertheless take place 25 years ago, and again it was caused not by a professional, but by an amateur interested in FLT with his conjecture corresponding to the restored sixth entry. Of course, to believe in all this is not easily, but also to invent such a thing is also hardly possible. Now we have to explain in more detail these restored entries in the margins and this will be done in the next points of our work and the same senator who started this whole story, will help us in this.

2. The History of Delusions

An unprecedented succession of failures, wrecks of secret hopes and defeats in the protracted for centuries storming of an impregnable fortress under name the Fermat's Last Theorem, turned into a such nightmare for science that even its very existence have been questioned. Like the fierce plague epidemic, the FLT not only deprived the minds of numerous amateur fermatists, scientists and unrecognized geniuses, but also very much contributed to the fact that the whole science was plunged into the abyss of uncontrollable chaos.

Pic. 12. Andrew Wiles



Already three and a half centuries have passed since the first publication of the FLT and twenty-five years after it was announced that in 1995 this problem was allegedly solved by Professor Princeton University USA Andrew Wiles.[6] However, once again it turned out this "epochal" event has nothing to do with the FLT![7] "The proof" of Wiles rests solely on the idea proposed by the German mathematician Gerhard Frey. This idea was rated as brilliant, but apparently only because that it was an elementary and even very common error!!!

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[6] It was a truly grandiose mystification, organized by Princeton University in 1995 after publishing in its own commercial edition "Annals of Mathematics" the "proof" of FLT by A. Wiles and the most powerful campaign in the media. It would seem that such a sensational scientific achievement should have been released in large numbers all over the world. But no! Understanding of this text is available only to specialists with appropriate training. Wow, now even that, which cannot be understood, may be considered as proof! However, for fairness it should be recognized that even such an overtly cynical mockery of science, presented as the greatest "scientific achievement" of the luminaries of Princeton University, cannot be even near to the brilliant swindle of their countrymen from the National Space Administration NASA, which resulted that the entire civilized world for half a century haven't any doubt that the American astronauts actually traveled to the moon!

[7] The "proof", which A. Wiles prepared for seven years of hard work and published on whole 130 (!!!) journal pages, exceeded all reasonable limits of scientific creativity and of course, him was awaiting inevitable bitter disappointment because such an impressive amount of casuistry understandable only to its author, neither in form nor in content is in any way suitable to present this as proof. But here the real wonder happened. Suddenly, the almighty unholy himself was appeared! Immediately there were influential people who picked up the "brilliant ideas" and launched a stormy PR campaign. And here is your world fame, please, many titles and awards! The doors to the most prestigious institutions are open! But such a wonder even for the enemy not to be wish because sooner or later the swindle will open anyway.

Pic. 13. Gerhard Frey

Instead of proving the impossibility of the Fermat equation $a^n+b^n=c^n$ in integers for $n>2$ here is proven only its incompatibility in the system with the equation $y^2=x(x-a^n)(x+b^n)$. In a similar way any nonsense can be proven. If the same work would be presented by one of the students, any of the professors would quickly bring him to clean water pointing to the obvious substitution of the subject of proof. Nevertheless, this super sensational news with great fanfare was noted in the world's leading media. The most influential newspaper of the USA "The New York Times" has been reported this right on the front page … in whole 2 years before the appearance of the "proof" itself!!! Andrew Wiles as the author of the "proof" became a member of the French Academy of Sciences and the laureate of as many as 18 of the most prestigious awards!!! To cover this momentous event, the British broadcaster BBC released an enthusiastic film and also it was invited the writer Simon Singh who published a book in 1997 titled "The Fermat's Last Theorem. The story of a riddle that confounded the world's greatest minds for 358 years".

Pic. 15. Simon Singh

Pic. 14. “The New York Times” of 06/24/1993 with an Article About Solving the FLT Problem

The New York Times

At Last, Shout of 'Eureka!' In Age-Old Math Mystery

SENATORS TAKE UP CLINTON'S BUDGET WITH ACID DEBATE

Split in Japan's Ruling Party Is Rearranging Political Map

House Retains Space Station In a Close Vote

Cutback in U.S. Funds to Reduce Summer Jobs for New York Youths

S.& L. Woes Impinge on Privileged Haven

Budget Debate: A Primer

U.S. Reports Setback On Russian Aid Plan

INSIDE

If Singh independently was preparing this book, then he would have so many questions that he would not have them managed for 20 years. Of course, he was helped in every way by the very heroes-professors having glorified in the BBC film, therefore the book became a success and it is really interesting to read it even to those who know about mathematics only by hearsay. The first thing that immediately catches your eye, is the fact that in the book it was made an arithmetic error (!) and not somewhere, but in its very name! Indeed, it is well known that "the greatest minds" could not know anything about the FLT before 1670 when its wording first appeared in a book published by Fermat's son Clément Samuel "Arithmetic" by Diophantus with comments by K. Bachet and P.

Fermat (see Appendix VI Pic. 96).[8] But then it should be not 358 but 325 years and it turns out that Singh simply did not notice the error?

However, don't rush to conclusions! This is not the book's author error and not at all accidental. These same professors vividly told Singh that supposedly back in 1637 [9] Fermat himself had noticed an error in his proof, but simply forgot to strike out recording of this theorem in the margins of the book. Who had invented this tale is unknown, but many scientists perceived it as a known fact and repeated time after time in their works. One can understand them because otherwise we could believe that Fermat turned out to be smarter than all of them! When Andrew Wiles said (https://www.pbs.org/wgbh/nova/article/andrew-wiles-fermat/):

"I don't believe Fermat had a proof" – this opinion was not new at all because many reputable scientists have repeated this many time. However, this is clearly against logic. It turns out that Fermat somehow managed to formulate an absolutely not obvious theorem without any reason whatsoever.[10]

Another contradiction in Singh's book is a clear discrepancy between the documentary facts and the assessments of Fermat as a scientist by consultants. It is necessary to pay tribute to Singh in that he is in good faith (although not fully) outlined that part of the Fermat's works, which relates to his contribution to science and is confirmed documental. Especially it should be noted that arithmetic is called in his book "the most fundamental of all mathematical disciplines". Only one listing of Fermat's achievements in science is enough to be sure that there were only a few scientists of such a level in the entire history of science.

But if this is so, then why was it necessary to think out something that is not confirmed by any facts and only distorts the real picture? This is very similar to the desire to convince everyone that Fermat could not prove the FLT since this is allegedly confirmed by historians. But historians received information from those mathematicians who did not cope with the Fermat's tasks and could in this way express their discontent. Hence, it's clear how appear all the arguments taken from nowhere that Fermat was an amateur scientist, arithmetic attracted him only with puzzles, which he "invented", FLT also was by him "invented" looking at the Pythagorean equation, and his proofs he did not want to publish because fear of criticism of colleagues.

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[8] If this book was published during the life of Fermat, then he would simply be torn to pieces because in his 48 remarks he did not give a proof of any one of his theorems. But in 1670 i.e. 5 years after his death, there was no one to punish with and venerable mathematicians themselves had to look for solutions to the problems proposed by him. But with this they obviously had not managed and of course, many of them could not forgive Fermat of such insolence. They were also not forgotten that during his lifetime he twice arranged the challenges to English mathematicians, which they evidently could not cope with, despite his generous recognition of them as worthy rivals in the letters they received from Fermat. Only 68 years after the first publication of Diophantus' "Arithmetic" with Fermat's remarks, did the situation at last get off the ground when the greatest science genius Leonard Euler had proven a special case of FLT for n=4, using the descent method in exact accordance with Fermat's recommendations (see Appendix II). Later thanks to Euler, there was received solutions also of the other tasks, but the FLT had so not obeyed to anyone.

[9] In pt. 2-30 of the letter Fermat to Mersenne, the task is set:"*Find two quadrate-quadrate, the sum of which is equal to a quadrate-quadrate or two cubes, the sum of which is a cube*" [9, 36]. The dating of this letter in the edition by Tannery is doubtful since it was written after the letters with a later dating. Therefore, it was most likely written in 1638. From this it is concluded that the FLT is appeared in 1637??? But have the FLT really such a wording? Even if these two tasks are special cases of the FLT, how it can be attributed to Fermat what about he could hardly even have guessed at that time? In addition, the Arabic mathematician Abu Mohammed al Khujandi first pointed to the insolubility of the problem of decomposing a cube into a sum of two cubes as early else the 10th century [36]. But the insolvability of the same problem with biquadrates is a consequence of the solution of the problem from pt. 2-10 of the same letter: "*Find a right triangle in numbers whose area would be equal to a square*." The way of proving Fermat gives in his 45th remark to Diophantus' "Arithmetic", which begins like this: "*If the area of the triangle were a square then two quadrate-quadrates would be given, the difference of which would be a square*." Thus, at that time, the wording of this problem and the approach to its solution were very different even from the particular case of FLT.

[10] In order no doubts to appear, attempts were made to somehow "substantiate" the fact that Fermat could not have the proof mentioned in the original of FLT text. See for example, https://cs.uwaterloo.ca/~alopez-o/math-faq/node26.html (Did Fermat prove this theorem?). Such an "argument" to any of the sensible people related to science, it would never come to mind because it cannot be convincing even in principle since in this way any drivel can be attributed to Fermat. But the initiators of such stuffing clearly did not take into account that this is exactly evidence of an organized and directed information campaign on the part of those who were interested in promoting Wiles' "proof".

That's what they really meant! Instead of the greatest scientist and founder of number theory as well as combinatorics (along with Leibniz), analytical geometry (along with Descartes), probability theory (along with B. Pascal), wave optics theory (along with Huygens), differential calculus (along with Leibniz and Newton), whose heritage was used by the greatest scientists in the course of centuries, suddenly a "lover" of puzzles appear, who only enjoyed the fact that no one could solve them. And since arithmetic is puzzles then this most fundamental of all sciences is relegated to the level of crosswords. Such a "logic" is clearly sewn with white threads and to be convinced of this, it is enough just to point out some well-known facts.

History has not retained any evidence that during the period life and activity of P. Fermat, someone has solved at least one of his tasks.[11] This fact became the basis for opponents else in those times to compose all kinds of tales about him. In the surviving letters, he reported that he had already sent proofs to his respondents three times. But none of these proofs reached us because Fermat's letters recipients in eyes of posterity of course, did not want to look like they could not cope with simple tasks. Another indisputable fact is that the Fermat's personal copy of the book "Arithmetic" by Diophantus edited in 1621 with his handwritten comments in the margins, none of the eyewitnesses have ever seen!!! Well, now just a most curious picture turns out. Fermat's critics seriously believe a witty Gascon joke that the Honorable Senator (apparently because of his lack of paper!) writes accurate and verified text of thirty-six Latin words in the book's margins, but are absolutely don't believe that he (the greatest scientist!) indeed had "truly amazing proof" of his own theorem.[12]

It is even difficult to imagine how these critics would have been amazed to find out that in fact Fermat had never dealt with the search for this proof since at that time he could not know what exactly is to be proven. But namely in the last sentence of the FLT wording, which had so much outraged them, there is a keyword directly indicated how he have solved this problem. It so happened that for centuries the science world vainly tormented itself in search of the FLT proof, but Fermat himself was never looked for it and simply had declared that he had it discovered![13]

It is possible also to remind to opponents ingeminating about Fermat's deliberate refusal to publish his works that for example, Descartes had received permission to publishing from Most Reverend cardinal Richelieu himself. It was impossible for Fermat and there is even a written (!!!) testimony about it (see text on P. Fermat's tombstone: "Vir ostentationis expers … – He was deprived the possibility of publication …". See Appendix VI Pic. 93 – 94). Nevertheless, even being in such conditions, he had prepared the publication of Diophantus' "Arithmetic" with the addition of his 48 comments, one of which got a name the "Fermat's Last Theorem".

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[11] An exception is one of the greatest English mathematicians John Wallis (see pt. 3.4.3).

[12] Obviously, if it come only about the wording of the FLT, it would be very unwise to write it in the margins of the book. But Fermat's excuses about narrow fields are repeated in other remarks for example, in the 45th, at the end of which he adds: "Full proof and extensive explanations cannot fit in the margins because of their narrowness" [36]. But only one this remark takes the whole printed page! Of course, he had no doubt that his Gascon humor would be appreciated. When his son, Clement Samuel who naturally found a discrepancy in the notes prepared for publication, was not at all surprised by this since it was obvious to him that right after reading the book it was absolutely impossible to give exact wording of tasks and theorems. The fact that this copy of Diophantus' "Arithmetic" with Fermat's handwritten notes didn't come to us suggests that even then this book was an extremely valuable rarity, so it could have been bought by another owner for a very high price. And he was of course not so stupid to trumpet about it to the whole world at least for his own safety.

[13] The text of the last FLT phrase: "*I have discovered a truly amazing proof to this, but these margins are too narrow to put it here*", obviously does not belong to the essence of the theorem, but for many mathematicians it looks so defiant that they tried in every way to show that it's just empty a Gascon boasting. At the same time, they did not notice neither humor about the margins nor the keyword "discovered", which is clearly not appropriate here. More appropriated words here could be, say, "obtained" or "founded". If Fermat's opponents paid attention to this, it would become clear to them that the word "discovered" indicates that he received the proof unexpectedly by solving the Diophantus' task, to which a remark was written called the FLT. Thus, mathematicians have unsuccessfully searched during the centuries for FLT proof instead of looking for a solution to the Diophantus' task of decomposing a square into the sum of two square. It seemed to them that the of Diophantus' task was clearly not worth their attention. But for Fermat it became perhaps the most difficult of all with it he has worked on, and when he did cope with it, then received the discovery of the FLT proof as a reward.

The publication was supposed to appear in honor of the historically significant event – the foundation of the French Academy of Sciences, in which preparation Fermat himself participated through the correspondence with his long-time colleague from the Toulouse parliament Pierre de Carcavy who became the royal librarian. The royal decree of the creation of the French Academy of Sciences was prepared by Carcavy and the all-powerful Finance Minister Jean-Baptiste Colbert submitting it to the signing by Louis XIV. However, the Academy of Sciences was established only in 1666 i.e. a year after the Fermat's death.

Mathematicians are very famous for how they are strict pedants, formalists and quibblers, but as soon as it comes to the FLT, all these qualities immediately disappear somewhere. Fermat's opponents ignoring well-known facts, called him either a hermit (this is a senator from Toulouse!) or a prince of amateurs (this is one of the founders of the French Academy of Sciences!), and this despite his contribution to science comparable to its importance only with a couple or triple of the most prominent scientists in the history of science!

They also did not fail sarcastically to point out that no one would have known about Fermat if the greatest mathematician of all times and peoples Leonhard Euler had not become interested in his tasks. But just this magic name has played a cruel joke with them. Their boundless belief in Euler's innovatory researches was too blind to notice that it was namely thanks to him science received such a powerful blow, from which it cannot recover up to now!

Mathematicians not only have believed Euler, but also warmly supported him that algebra is the main mathematical science, while arithmetic is only one of its elementary sections.[14] Euler's idea was really excellent because his algebra, which gained new possibilities through the use of "complex numbers" was to be a most powerful scientific breakthrough that would allow not only to expand the range of numbers from the number axis to the number plane, but also to reduce the most of all calculations to solving algebraic equations. [15]

The need for "complex numbers" mathematicians explained very simply. To solve absolutely any algebraic equations, you need (not so much!) to make the equation $x^2 + 1 = 0$ become solvable. [16] In Russian this is called: "Don’t sew the tail to a mare"! This equation is not at all harmless since it has nothing to do with practical tasks, but undermines the fundamentals of science very substantially. Nevertheless, the devilish temptation to create something very spectacular on empty place turned out to be stronger than common sense and Euler decided to demonstrate the new mathematical possibilities in practice.

Pic. 16. Leonhard Euler

[14] It is curious that the Russian-language edition this fundamental work of Euler was published in 1768 under the title "Universal Arithmetic" although the original name "Vollständige Anleitung zur Algebra" should be translated as the Complete Introduction to Algebra. Apparently, translators (students Peter Inokhodtsev and Ivan Yudin) reasonably believed that the equations are studied here mainly from the point of view of their solutions in integers or rational numbers i.e. by arithmetic methods. For today's reader this 2-volume edition is presented as a Chinese literacy because along with the highly outdated Russian language and spelling, there is simply an incredible number of typos. It is unlikely that today's RAS as the heiress of the Imperial Academy of Sciences, which published this work, understands its true value, otherwise it would have been reprinted a long time ago in a modern and accessible form.

[15] Here there is an analogy between algebra and the analytic geometry of Descartes and Fermat, which looks more universal than the Euclidean geometry. Nevertheless, Euclidean arithmetic and geometry are the only the foundations, on which algebra and analytical geometry can appear. In this sense, the idea of Euler to consider all calculations through the prism of algebra is knowingly flawed. But his logic was completely different. He understood that if science develops only by increasing the variety of equations, which it is capable to solve, then sooner or later it will reach a dead end. And in this sense, his research was of great value for science. Another thing is that their algebraic form was perceived as the main way of development, and this later led to devastating consequences.

[16] Just here is the concept of a “number plane” appears, where real numbers are located along the x axis, and imaginary numbers along the y axis i.e. the same real, only multiplied by the “number” $i = \sqrt{-1}$. But along that come a contradiction between these axes – on the real axis, the factor 1^n is neutral, but on the imaginary axis no, however this does not agree with the basic properties of numbers. If the “number” *i* is already entered, then it must be present on both axes, but then there is no sense in introducing the second axis. So, it turns out that from the point of view of the basic properties of numbers, the ephemeral creation in the form of a number plane is a complete nonsense.

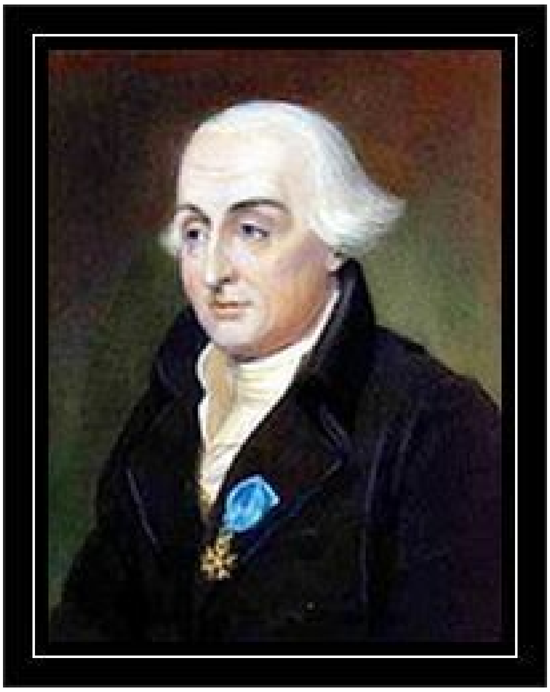
The FLT, which Euler could not to prove, would be perfect for demonstrating the possibilities of a new wonder-algebra. However, the result turned out to be more than modest. Instead of a general proof of FLT, only one particular case for the 3rd power was proven [8, 30]. More ambitious was seemed the proof of other Fermat's theorem about the only solution in integers of the equation $y^3 = x^2 + 2$ [36] because it was a very difficult task and like FLT, none of the mathematicians could solve it. Despite the fact that the very possibility of solvability of any algebraic equation has not yet been proven, these Euler's demonstrations were perceived by hurrah. It only remained to find a solution to the problem called the “Basic Theorem of Algebra”. In 1799 the real titan of science Carl Gauss coped brilliantly with this task presenting proof even in 4 different ways!

The scientific community greeted all these "achievements" with a storm of applause while the unholy was also so glad that it is impossible to imagine.

Pic. 17. Karl Friedrich Gauss

Yeah, this was need to be seen how the whole civilized scientific world has driven itself into a dead end! It is obvious that for science, which does not rely on arithmetic, there are no reasonable restrictions and the consequences will be sad – from the dominance of algebra, arithmetic will become so difficult that witlings will call it a science for the elitist mathematicians where they can demonstrate the sharpness of their mind! But the scientists themselves unsuspecting and full of the best intentions, continued to advance science forward to new heights, but so diligently that they either inadvertently or due to a misunderstanding… simply have lost the Fermat's Golden Theorem (FGT)! But this was one of the most impressive discoveries of Pierre Fermat in arithmetic, of which he was very proud.

Pic. 18. Joseph Lagrange



It was so happened that the third in the history royal mathematician Joseph Lagrange together with his predecessor the second royal (and the first imperial!) mathematician Leonard Euler, have proven in 1772 only one special case of FGT for squares and became famous for all the world. This remarkable achievement of science was called the “Lagrange's Theorem about Four Squares”. Probably it is good that Lagrange didn’t live after two years until the moment when in 1815 still very young Augustin Cauchy presented his general proof of the FGT for all polygonal numbers. But then suddenly something terrible happened, the unholy appeared from nowhere and put his "fe" in. And here isn't to you any world fame and besides, you get complete obstruction from colleagues.

Pic. 19. Augustin Cauchy

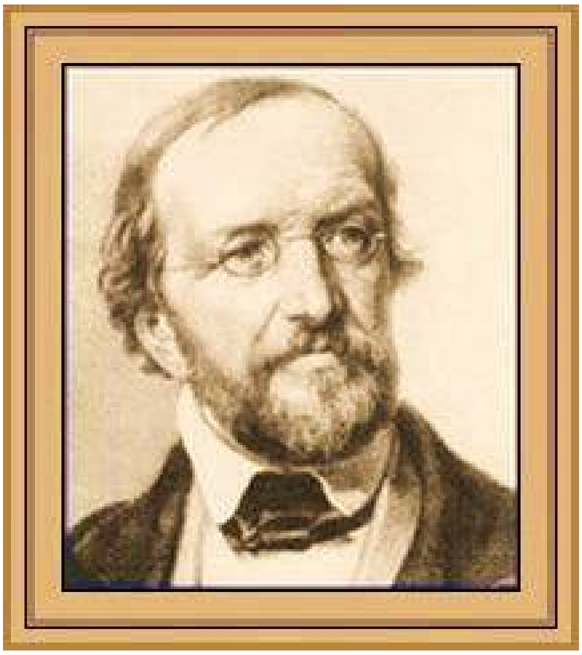
Well, nothing can be done here, academicians did not like Cauchy and they achieved that this general proof of the FGT was ignored and did not fall into the textbooks as well as no one remembers the Gauss' proofs of 1801 for triangles and for the same squares, nevertheless in the all textbooks as before and very detailed the famous Lagrange's theorem is given. However, after Google published a facsimile of the Cauchy proof of FGT [3], it became clear to everyone why it was not supported by academics (see pt. 3.4.2).

Pic. 20. Marie-Sophie Germain

In the meantime, scientists from around the world inspired by these grand shifts, have so perked up that they wanted overcome the very FLT! They were joined by another famous woman very well known among scientists and mathematicians Marie-Sophie Germain. This talented and ambitious Mademoiselle proposed an elegant way, which was used by at once two giants of mathematical thought Lejeune Dirichlet and Adrien Legendre to prove … only one special case of FLT for the fifth power.

Another such giant Gabriel Lame managed to do the almost impossible and get proof of the highest difficulty … of another particular case of FLT for the seventh power. Thus, the whole elitist quad of the representatives from the high society of scientists was able to prove whole two (!) particular cases of FLT [6, 38].

Pic. 21. Lejeune Dirichlet



Pic. 22. Adrien Legendre

Pic. 23. Gabriel Lame

This result could have been proud since even Euler was also able to prove only two particular cases of FLT for 3rd and 4th powers. In the proof for the 4th power he has applied the descent method following exactly the recommendations of Fermat (see Appendix II). This case is especially important because its proof is valid for all even powers i.e. to obtain a general proof of FLT only odd powers can be considered.

It should be noted that namely Euler has solved (and even significantly expanded!) almost all the most difficult Fermat's tasks and if not for him, then the name Fermat alone could cause real chills to mathematicians. But just not to Sophie Germain who was not at all satisfied the situation with the unproven FLT and she even ventured to suggest that Gauss himself should take up this task! But he simply waved away her replying that the FLT is of little interest to him and such statements, which can neither be proven nor refuted can be found as many as you like.

Of course, Gauss himself would be happy to serve this lady, but if he could do this then it would not need to persuade him. For example, with the help of the "Deductions' arithmetic" developed by Gauss, the prototype of which was the "The Fermat's little theorem", it was clearly shown how may be to solve the most difficult problems of arithmetic effectively. In particular, only Gauss managed to find a solution to the Fermat's task of calculating two the only squares, the sum of which is a given prime number of types 4n+1 [11, 25].

A characteristic feature of Gauss is his dislike for dubious innovations. For example, he could hardly imagine himself the creator of the geometry of curved spaces. But when he established that such geometry could take place and not contain contradictions, he was very puzzled by this. He was sure that his find could not be of practical use due to the absence of any real facts confirming something like that. However, he quickly found a good way out – he just helped to publish this discovery to his Russian colleague Nikolai Lobachevsky and have done it so skillfully that no one was even surprised when a Russian professor and rector of Kazan University have published a work on non-Euclidean geometry … in Berlin and in German! In the future, Gauss' doubts were confirmed. Followers appeared and flooded science with a whole bunch of similar "discoveries".

Despite the fact that with his proof of the "Basic Theorem of Algebra" Gauss supported Euler in promoting his idea of using "complex numbers", he did not find any other opportunities for progress in this direction. And what Euler showed, he was also not impressed. Moreover, even modern science at all can nothing offer anything on the use of “complex numbers”. But the sea of all kinds of “scientific” works, studies and textbooks on this theme is clearly inadequate with its true value. Gauss felt that something was amiss with these “numbers” and that it would not end well, therefore in that direction he did not work.

Pic. 24. Ernst Kummer

Thunder struck in 1847 when at a meeting of members of the French Academy of Sciences Gabriel Lame and Augustin Cauchy reported that their FLT proofs was ready for consideration at the competition. However, when in order to identify the winner, it was already possible to open received from them the sealed envelopes, the German mathematician Ernst Kummer having put all scientists on the sinful earth. In his letter it was reported that the FLT proof on the basis of “complex numbers” is impossible due to the ambiguity of their decomposition into prime factors.[17]

[17] According to the Basic theorem of arithmetic the decomposition of any natural number into prime factors is always unambiguous, for example, 12=2×2×3 i.e. with other prime factors this number like any other, is impossible to imagine. But for “complex numbers” in the general case this unambiguity is lost for example, $12=(1+\sqrt{-11})\times(1+\sqrt{-11})=(2+\sqrt{-8})\times(2+\sqrt{-8})$ In fact, this means the collapse of science in its very foundations. However, the generally accepted criteria (in the form of axioms) what can be attributed to numbers and what is not, as there was not so still is not.

Here you have got what you want! These very “complex numbers” are not any numbers!!! And one could notice finally, after arithmetic was knocked from under science, it hangs in the air having no solid foundation. And the mistakes of the greats in their consequences are also extreme and they begin to break down a science so much that, instead of a holistic system of knowledge, it creates a bunch of unrelated fragments.

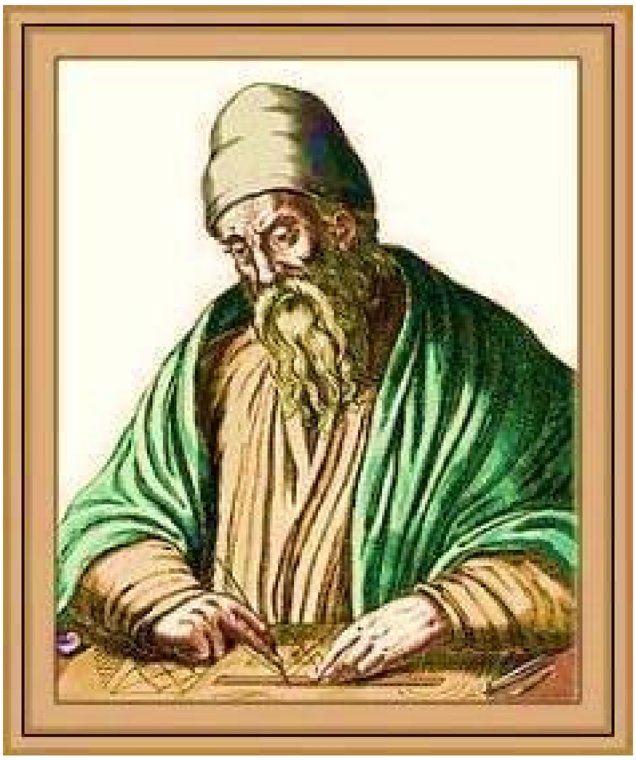
If it so happened, then else in 1847, these very “Complex numbers” had to be solemnly buried with all the honors. But with this matter somehow did not work out at all and the restless souls of the long-dead theories turn out to be so tenacious that they cannot be expelled from textbooks and professorial lectures by any means. They will wander through different books and reference books whose authors will be completely unaware of how much their works depreciates from this useless ballast.

In the mentioned book of Singh is well shown as the ambiguity of decomposing compound integers into prime factors makes it impossible to construct logical conclusions in proofs and it also was said that the unambiguity theorem for such a decomposing for natural numbers was given in “Euclidean Elements”. The specific book and location of the theorem is not specified; therefore, it is rather difficult to find the necessary text, but it really turned out to be so.[18]

“Euclidean Elements" is a very old book with archaic terminology, in which this extremely important for science theorem was somehow lost and it was simply forgotten about it. The first to discover the loss was Gauss. He formulated it again and gave proof, which contained a surprisingly simple and even childish error, where as an argument used exactly what needs to be proven (see pt. 3.3.1).

But this is not an ordinary theorem, all science holds on it! And what about Euclid? Oh my God! In fact, his proof is the same as that of Gauss i.e. wrong!!! Tell it to someone, so they will not believe! Three giants of science are stumbled on the same place!

Pic. 25. Euclid



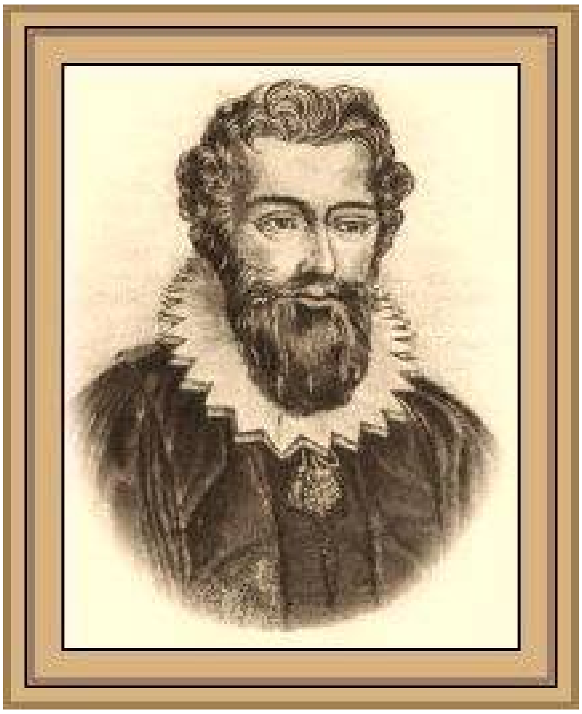
[18] The theorem and its proof are given in “The Euclid's Elements” Book IX, Proposition 14. Without this theorem, the solution of the prevailing set of arithmetic problems becomes either incomplete or impossible at all.

Then it turns out that this whole science is fake and now, thanks to Singh's book and despite all the good intentions of its author, this terrifying FLT, which now even in theory has become completely unprovable, was so furious that like a true monster, in one fell swoop have devalued all the age-old works of scientists! And yet they live in not fabulous, but in the real kingdom of crooked mirrors, what about they themselves don't know anything.

The fiasco being by academicians Cauchy and Lame did not result in the rejection use of the surrogates of numbers in science especially after Kummer who had crushed their works, found a way to prove FLT (with a little modernization) for any particular case. Before the final victory over the FLT only a last step remained – to obtain a single common proof. Since then 170 years have passed, but nothing was changed. Supported in due time by the Euler's genius "complex numbers" are still presented today as a kind of extension the notion of number. This looks very impressive and solid, but still requires a clear definition of the very notion of number, however just with this deal are very bad.

Students intuitively feeling that they are being tortured in vain by nonsenses about some non-existent numbers, suddenly have a question: "What is a number?" They never come to mind that not a single professor could not answer this question even if he has reread everything that is in mathematics. One of them even could not bear the mocking hints and had published a whole book called "What is a number?" [13, 29]. In it, he has written so many whatnots that students have very well understood – such a question it's better not to ask.

Pic. 26. Francis Viète

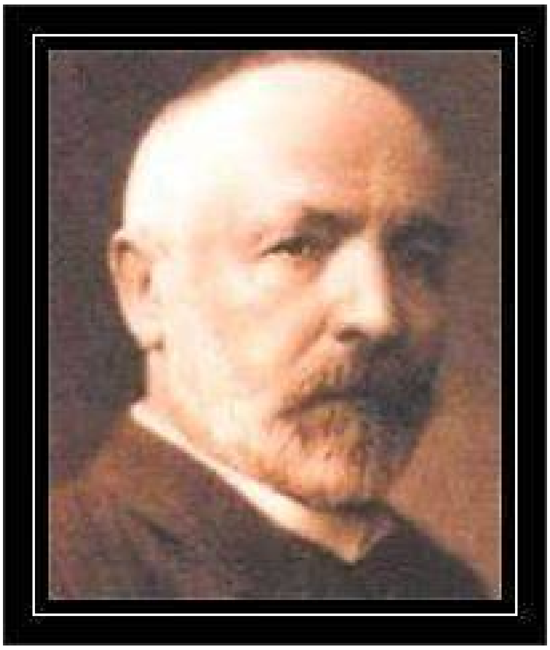


Meanwhile, scientists continued to move science forward, not bothering with such trifles as the essence of the notion of number. So, they created a whole bunch of new algebras taking advantage of the fact that there were no obstacles along the way. But they were not a continuation of what was a real one, the founder of which was the first royal mathematician François Viète served as an advisor to the court of the French king Henry III. But if these new algebras are special, then their terminology and bases are also special.

So, little by little in the science began to form a particular bird language understandable only to the authors of these most innovative developments. It even reached the point where mathematical societies creating a science only for themselves began to appear and in addition to this, the newest numbers appeared out of nothing: “hypercomplex”, “quaternions”, “octonions” etc. But the impression from the new achievements sometimes was spoiled from the same mare tail,[19] which from somewhere appeared again. Getting this tail in the face is not very pleasant, but this is already the costs of a profession. In an effort to get away from such costs, a brilliant way out of the difficulties with the definition the essence the notion of a number was found. Scientists have finally grasped that it needs to be derived from other simpler notions, for example, such as the notion “set”. Everything turned out so simple, a set is that what is a lot. Well, is it not clear? However, it was found out again that one cannot do without empty set and in this case, it would see like nothing, and the question again arises, so what is a set number or not?

Georg Cantor has developed his theory of sets, which other mathematicians such as, for example, Henri Poincaré, called all sorts of bad words and did not want to admit at all. But suddenly unexpected for everyone the respectable "Royal Society of London" (the English Academy of Sciences) in 1904 decided to award Cantor with its medal. So, it turns out that here is the point, where the fates of science are decided![20]

Pic. 27. Georg Cantor



And everything would be fine, but suddenly another trouble struck again. Out of nowhere in this very theory of sets insurmountable contradictions began to appear, which are also described in

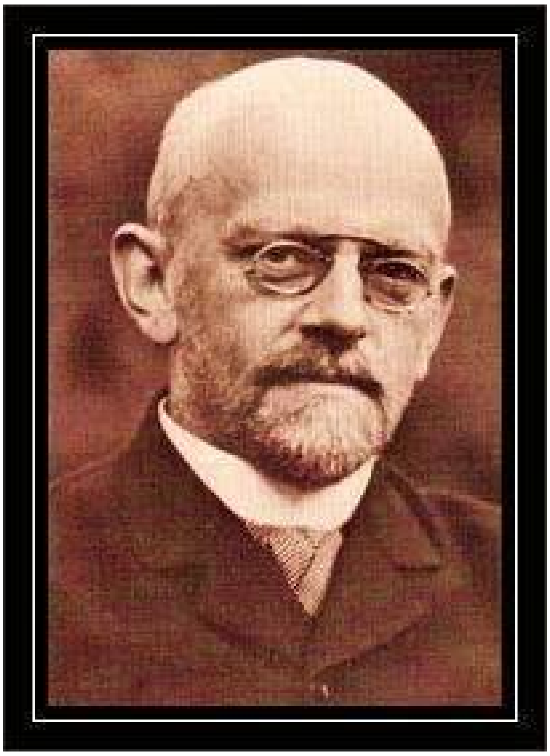
[19] Soviet mathematician Lev Pontryagin showed these “numbers” do not have the basic property of commutativity i.e. for them ab ≠ ba [34]. Therefore, one and the same such “number” should be represented only in the factorized form, otherwise it will have different value at the same time. When in justification of such creations scientists say that mathematicians have lack some numbers, in reality this may mean they obviously have lacked a mind.

[20] If some very respected public institution thus encourages the development of science then what one can object? However, such an emerging unknown from where the generosity and disinterestedness from the side of the benefactors who didn’t clear come from, looks somehow strange if not to say knowingly biased. Indeed, it has long been well known where these “good intentions” come from and whither they lead and the result of these acts is also obvious. The more institutions there are for encouraging scientists, the more real science is in ruins. What is costed only one Nobel Prize for "discovery" of, you just think … accelerated scattering of galaxies!!!

great detail in Singh's book. In the scientific community everyone immediately was alarmed and began to think about how to solve this problem. But it has rested as on the wall and in no way did not want to be solved. Everyone was somehow depressed, but then they yet cheered up again.

It was so happened because now David Hilbert himself got down to it, the great mathematician that first solved the very difficult Waring problem, which has a direct relationship to the FLT. [21] It is also curious that Hilbert repeated Euler's experiment apparently inspired by the FLT problem. It seems that at some point Euler began to have doubts that the FLT is generally provable and he assumed the equation $a^4+b^4+c^4=d^4$ also like Fermat's equation $a^n+b^n=c^n$ for n>2 in integers is unsolvable, but in the end it turned out that he was wrong.[22]

Pic. 28. David Hilbert



Following the example of Euler on the eve of the 20th century, Hilbert offered to the scientific community 23 problems, which according to his assumption, are unlikely to be solved in the foreseeable future. Nevertheless, Hilbert's colleagues coped with them rather quickly, while Euler's hypothesis has held almost until the 21st century and was only refuted with the help of computers, what is also described in Singh's book. So, the suspicion that the FLT was merely an assumption of its author has lost any reason.

Hilbert had not cope with overcoming contradictions in set theory and could not do it because this problem is not at all mathematical, but informational one, so computer scientists should solve it

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[21] Waring's problem is the statement that any positive integer N can be represented as a sum of the same powers x_i^n, i.e. in the form $N = x_1^n + x_2^n + \dots + x_k^n$. It was in very complex way first proven by Hilbert in 1909, and in 1920 the mathematicians Hardy and Littlewood simplified the proof, but their methods were not yet elementary. And only in 1942 the Soviet mathematician Yu. V. Linnik has published arithmetic proof using the Shnielerman method. The Waring-Hilbert theorem is of fundamental importance from the point of view the addition of powers and does not contradict to FLT since there are no restrictions on the number of summands.

[22] A counterexample refuting Euler's hypothesis is $95800^4 + 217519^4 + 414560^4 = 422481^4$. Another example $2682440^4+15365639^4+18796760^4=20615673^4$. For the fifth power everything is much simpler. $27^5+84^5+110^5+133^5=144^5$. It is also possible that a general method of such calculations can be developed if we can obtain the corresponding constructive proof of the Waring's problem.

sooner or later and when this happened, they are surprisingly very easily (and absolutely true) found a solution just forbidding closed chains of links.[23] It is clear Hilbert could not know about it then and decided that the most reliable barrier to contradictions can be provided with the help of axioms. But axioms cannot be created on empty place and must come out of something and this something is a number, but what it is, no one can explain this not then nor now.

A brilliant example of what can be created with axioms is given in the same book of Singh. The obvious incident with the lack of a clear formulation to the notion of a number can accidentally spoil any rainbow picture and something needs to be done with it. It gets especially unpleasant with the justification of the “complex numbers”. Perhaps this caused the appearance in the Singh’s book of Appendix 8 called “Axioms of arithmetic”, in which 5 previously known axioms relating to a count are not mentioned at all (otherwise the idea will not past), while those that define the basic properties of numbers are complemented and a new axiom appears so that it must exist the numbers n and k, such that n+k=0 and then everything will be in the openwork!

Of course, Singh himself would never have guessed this. It is clearly visible here the help of consultants who for some reason forgot to change the name of the application since these are no longer axioms of arithmetic because already nothing is left of it.[24] The school arithmetic, which for a long time barely kept on the multiplication table and the proportions, is now completely drained. Instead it, now there is full swing mastering of the calculator and computer. If such “progress” continues further, then the transition to life on trees for our civilization will occur very quickly and naturally.

Against this background a truly outstanding scientific discovery was made in Wikipedia, which simply has no equal in terms of art and the scale of misinformation. For a long time, many people thought that there are only four actions of arithmetic, these are addition and subtraction, multiplication and division. But no! There are also exponentiation and … root extraction (???). The authors of the articles given us this "knowledge" through Wikipedia clearly blundered because extracting the root is the same exponentiation only not with the integer power, but with fractional one. No of course, they knew about it, but what they didn’t guess was that it was they who copied this arithmetic action at Euler himself from that very book about the wonder-algebra[25].

The correct name of the sixth action of arithmetic is logarithm i.e. calculating the power index (x) for a given power number (y) and basis of a power (z) i.e. from $y=z^x$ follows $x=\log_z y$. As in the case with the name of the Singh’s book, this error is not at all accidental since no one really worked on logarithms as part of the arithmetic of integers. If this happens someday, then not earlier than in some five hundred years! But as for the action with power numbers, the situation here is not much better than with logarithms. If multiplication and dividing of power numbers as well as exponentiation a power number to a power, do not present any difficulties, but the addition of power numbers is still a dark forest even for professors.

The clarification in this matter begins with the FLT, which states that the sum of two power integer with the same power index greater than the second, cannot be an integer with the same power index. In this sense, this theorem is not at all any puzzle, but one of the basic propositions that unequivocally (!) regulates the addition of integer powers, therefore, it is of fundamental importance

[23] Of course, this does not mean that computer scientists understand this problem better than Hilbert. They just had no choice because closed links are looping and this will lead to the computer freezing.

[24] The axiom that the sum of two positive integers can be equal to zero is clearly not related to arithmetic since with numbers that are natural or derived from them this is clearly impossible. But if there is only algebra and no arithmetic, then also not only a such things would become possible.

[25] It is curious that even Euler (apparently by mistake) called root extraction the operation inverse to exponentiation [8], although he knew very well that this is not so. But this is no secret that even very talented people often get confused in very simple things. Euler obviously did not feel the craving for the formal construction of the foundations of science since he always had an abundance of all sorts of other ideas. He thought that with the formalities could also others coped, but it turned out that it was from here the biggest problem grew.

for science.[26] The fact that the FLT has not yet been proven, indicates only the state of current science, which is falling apart right before our eyes. Science cannot even imagine that if the proof from Fermat himself came to us, it would have been long ago taught in secondary school.

Many people of course, will perceive it as a fairy tale, but only the completely blind ones may not notice that behind all this absurd and awkward history with the FLT, clearly and openly ears of the unholy stick so out, that he was enough to deprive human civilization of access to Fermat's works on arithmetic, so it immediately turned out to be completely disoriented. Instead of developing science they began being vigorously to destroy it and even with very good intentions. But a special zeal in people appears when they have the material stimulus.

Pic. 29. Andrew Beal

Texas entrepreneur Andrew Beal[27] had proposed his conjecture, the proof of which allegedly could lead to a very simple proof of the FLT. Since for the solution of this problem it was proposed first $ 5 thousand, then $ 100 thousand, and from 2013 – a whole million, then naturally it appeared many willing people who began diligently this task to solve. However, in the conditions when arithmetic has long ceased to be the primary basic of all knowledge and still does not know, what is a number, everything turned upside down i.e. one amateur enthusiast was able to set on the ears the whole official science and so, that it had in fact already acknowledged the experience of Baron Munchhausen lifting himself up, taking himself by his collar, wherewith science did not even try at least to conceal its own insolvency (see pt. 4.5).

By working in the intense and tireless search for the FLT proof, it has never even occurred to anyone to search for Fermat's manuscripts with layouts and calculations, without which he could not do[28]. However, again from Singh's book we learn that such an idea came to Euler who asked his friend living in Lausanne (a city not far from Toulouse) to look for at least a little piece of paper with

[26] This is evident at least from the fact, in what a powerful impetus for the development of science were embodied countless attempts to prove the FLT. In addition, the FLT proof, obtained by Fermat, opens the way to solving the Pythagorean equation in a new way (see pt. 4.3) and magic numbers like $a+b-c=a^2+b^2-c^2$ (see pt. 4.4).

[27] In the Russian-language section of Wikipedia, this topic is titled "Гипотеза Била". But since the author's name is in the original Andrew Beal, we will use the name of the "Гипотеза Биэла" to avoid confusion between the names of Beal (Биэл) and Bill (Бил).

[28] In a letter from Fermat to Mersenne from 06/15/1641 the following is reported: "*I try to satisfy Mr. de Frenicle's curiosity as completely as possible … However, he asked me to send a solution to one question, which I postpone until I return to Toulouse, since I am now in the village where I needed would be a lot of time to redo what I wrote on this subject and what I left in my cabinet*" [9, 36]. This letter is a direct evidence that Fermat in his scientific activities could not do without his working recordings, which, judging by the documents reached us, were very voluminous and could hardly have been kept with him on various trips.

Fermat’s instructions to the FLT proof. But nothing was found, however, they were looking for what we do not really need! It was necessary to look for a cache!!!

Here is the new puzzle, which is not easier! What else kind of cache? … Oh yes! The fact is that only those Fermat's works remained, which he itself had already prepared for publication since otherwise they would hardly have been published. But all the working manuscripts for some reason has disappeared. It looks very strange and it is possible that they can still be kept in the cache, which Fermat has equipped to store the material evidence necessary for him to work as a senator and high-ranking judge. It was quite reasonable to keep calculations and proofs there, since Fermat’s scientific achievements could significantly damage his main work if they were made public before the establishment of the French Academy of Sciences.[29]

If we could somehow look into this cache, what will we see there? To begin with, let's try to find some simple tasks there. For example, the one that Fermat could offer today for secondary school students:

Divide the number x^n-1 *by the number* $x-1$*, or the number* $x^{2n}-1$ *by the number* $x\pm1$*, or the number* $x^{2n+1}+1$ *by the number* $x+1$.

It is obvious that students with the knowledge of solving such a task will be simply a head over the current students who are trained in the methods of determining the divisibility by only some small numbers. But if they else know a couple of the Fermat's theorems, they can easily solve also the more difficult problem:

Find two pairs of squares, each of which adds up to the same number in the seventh power, for example,
$221^7=151114054^2+53969305^2=82736654^2+137487415^2$

Compared to the previous task where calculations are not needed at all, in solving this task, even with a computer calculator you have to tinker with half an hour to achieve a result, while apart from understanding the essence of the problem solution, you need to show a fair amount of patience, perseverance and attention. And who understands the essence of the solution, will be able to find other solutions to this problem.[30]

Of course, such tasks can cause a real shock to today's students and especially to their parents who will even demand not to “dry the brains” of children. But if children's brains are not filled with elementary knowledge and not trained by solving the corresponding tasks, they will wither by themselves. This is proven by the statistics of the steady decline in today's students IQ compared with

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[29] If Fermat would live to the time when the Academy of Sciences was established and would become an academician then in this case at first, he would publish only problem statements and only after a sufficiently long time, the main essence of their solution. Otherwise, it would seem that these tasks are too simple to study and publish in such an expensive institution.

[30] To solve this problem, you need to use the formula that presented as the identity: $(a^2+b^2)\times(c^2+d^2)=(ac+bd)^2+(ad-bc)^2=(ac-bd)^2+(ad+bc)^2$. We take two numbers 4 + 9 = 13 and 1 + 16 = 17. Their product will be $13\times17 = 221 = (4 + 9) \times (1+16) = (2\times1 + 3\times4)^2 + (2\times4 - 3\times1)^2 = (2\times1 - 3\times4)^2 + (2\times4 + 3\times1)^2 = 14^2 + 5^2 = 10^2 + 11^2$; Now if $221^6 = (221^3)^2 = 10793861^2$; then the required result will be $221^7 = (14^2 + 5^2)\times10793861^2 = (14\times10793861)^2 + (5\times10793861)^2 = 151114054^2 + 53969305^2 = (10^2 + 11^2)\times10793861^2=(10\times10793861)^2 + (11\times10793861)^2=107938610^2 + 118732471^2$; But you can go also the other way if you submit the initial numbers for example, as follows: $221^2 = (14^2 + 5^2)\times(10^2 + 11^2) = (14\times10 + 5\times11)^2 + (14\times11 - 5\times10)^2 = (14\times10 - 5\times11)^2 + (14\times11+5\times10)^2 = 195^2 + 104^2 = 85^2 + 204^2$; $221^3 = 221^2\times221 = (195^2 + 104^2)\times(10^2 + 11^2) = (195\times10 + 104\times11)^2 + (195\times11 - 104\times10)^2 = (195\times10 - 104\times11)^2 +(195\times11 + 104 \times 10)^2 = 3\,094^2 + 1105^2 = 806^2 + 3185^2$; $221^4 = (195^2 + 104^2)\times(85^2 + 204^2) = (195\times85 + 104\times204)^2 + (195\times204 - 85\times104)^2 = (195\times85 - 104\times204)^2 + (195\times204 + 85\times104)^2 = 37791^2 + 30940^2 = 4641^2 + 48620^2$; $221^7 = 221^3\times221^4 = (3094^2 + 1105^2)\times(37791^2 + 30940^2) = (3094\times37791 + 1105\times30940)^2 + (3094\times30940 - 1105\times37791)^2 = (3094\times37791 - 1105\times30940)^2 + (3094\times30940 + 1105\times37791)^2$; $221^7 = 151114054^2 + 53969305^2 = 82736654^2 + 137487415^2$

their predecessors. Really in fact, the above tasks are only a warm-up for the young generation, but children could produce a real furor for mathematicians offering them some simple Fermat's theorems about magic numbers (see Pt. 4.4.). And this is else a big question, could these theorems being solved by today's professors or will they again need some three hundred years and the story with the FLT will repeat? However, the chances of them in contrast to previous times, are very high because magic numbers are a direct consequence of the same “truly amazing” proof of the FLT, about the existence of which we have direct written evidence from Fermat himself.

Reconstruction of this proof was briefly published as early as 2008 [30], but the unholy was on the alert and presented this event so, that modern science disoriented by the false notion that the problem was solved long ago, has not paid on this any attention. However, all secret sooner or later becomes clear and the decisive word in spite of everything, still remains for science. The question now is only when this science will finally awaken and comes to his senses. The longer it will be in a blissful state of oblivion, the sooner the terrible events will come that already now beginning to shake our world like never before.

In order for science to win a well-deserved victory over the gloom of ignorance and mass disinformation, which are triumphant today, it needs very little. For the beginning it is necessary simply to search for the very cache, in which such secrets of science are hidden, that have not lost their relevance for three and a half centuries.[31] Even if the papers found in the cache will be unreadable, the very fact of the existence of the cache will be evidence that science is moving in the right direction and the results will not be long in coming.

We already did something in this direction when we restored the FLT recording in the margins of Diophantus 'Arithmetic' (see pic. 5 and the translation in the end of Pt. 1). Now, by all means, we need to get a complete picture of the whole sequence of events that led to the discovery of the FLT in its final wording published in 1670. It will not be easily at all, but since we got involved in this story, now we have nowhere to retreat and we will strain all our forces to achieve the aim. Fortunately, for this we have all the opportunities granted to us from above to get the coveted access to the cache of the Toulousean senator.

[31] If Fermat's working notes were found, it would turn out that his methods for solving tasks are much simpler than those that are now known, i.e. the current science has not yet reached the level that took place in his lost works. But how could it happen that these recordings disappeared? There may be two possible versions. The first version is being Fermat’s cache, which no one knew about him. If this was so, there is almost no chance it has persisted. The house in Toulouse, where the Fermat lived with his family, was not preserved, otherwise there would have been a museum. Then there remain the places of work, this is the Toulouse Capitol (rebuilt in 1750) and the building in the city of Castres (not preserved) where Fermat led the meeting of judges. Only ghostly chances are that at least some walls have been preserved from those times. Another version is that Fermat’s papers were in his family’s possession, but for some reason were not preserved (see Appendix IV, year 1660, 1663 and 1680).

3. What is a Number?

3.1. Definition the Notion of Number

The question about the essence the notion of number at all times was for scientists the thing-in-itself. They of course, understood that they could not distinctly answer this question as well as they could not admit in this since this would have a bad effect on maintaining the prestige of science. What is the problem here? The fact is that in all cases a number must be obtained from other numbers, otherwise it cannot be perceived as a number. To understand for example, the number 365, you need to add three hundred with six tens and five units. It follows that the notion of a number does not decompose into components that are qualitatively different from it and in such a way as usual for science i.e. through analysis, it is not possible to penetrate the secret of its essence.

Scientists having a question about the nature of numbers immediately ran into this problem and came to the conclusion that a general definition the notion of number simply does not exist. But not a such was Pierre Fermat who approached this problem from other side. He asked: "Where does the notion of number come from?" And came to the conclusion that his predecessors were the notions "more", "less" and "equal" as the comparisons' results of some properties inherent to different objects [30].

If different objects are compared in some property with the same object then such a notion as a measurement appears, so perhaps is the essence of a number possible revealed through a measurement? However, it is not so. In relation to the measurement, the number is primary i.e. if there are no numbers, there can be no also measurements. Understanding the essence of the number becomes possible only after establishing the number is inextricably connect with the notion of "function".

But this notion is not difficult to determine:

A function is a given sequence of actions with its arguments.

In turn, actions cannot exist on their own i.e. in the composition of the function in addition to them must include the components, with which these actions are performed. These components are called *function arguments*. From here follows a general definition the notion of number:

Number is an objective reality existing as a countable quantity, which consists of function arguments and actions between them.

For example, a+b+c=d where a, b, c are arguments, d is a countable quantity or the *number value*.[32]

To understand what a gap separates Pierre Fermat from the rest of the science's world, it is enough to compare this simple definition with the understanding existing in today's science [13, 29]. But understanding clearly presenting in the scientific works of Fermat, allowed him still in those distant times to achieve results that for other scientists were either fraught with extreme difficulties or even unattainable. It may be given also the broader definition the notion of number, namely:

A number is a kind of data represented as a function.

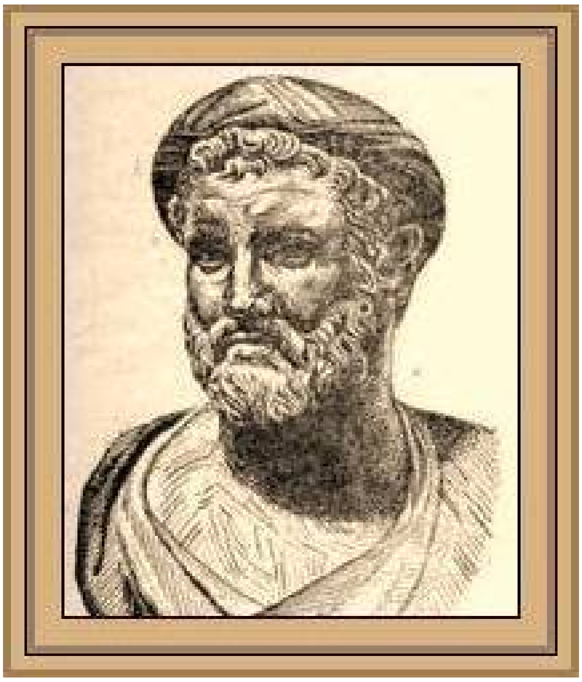
This extended definition the notion of number goes beyond frameworks mathematics; therefore, it can be called as general one and the previous definition as mathematical. In this second

[32] For mathematicians and programmers, the notion of function argument is quite common and has long been generally accepted. In particular, f (x, y, z) denotes a function with variable arguments x, y, z. The definition of the essence of a number through the notion of function arguments makes it very simple, understandable and effective since everything what is known about the number, comes from here and all what this definition does not correspond, should be questioned. This is not just the necessary caution, but also an effective way to test the strength of all kinds of structures, which quietly replace the essence of the number with dubious innovations that make science gormlessly and unsuitable for learning.

definition, it is necessary to clarify the essence the notion of “data”, however, for modern science this question is no less difficult than the question about the essence of the notion a number.[33]

From the general definition the notion of number follows the truth of the famous Pythagoras' statement that everything existing can be reflected as a number. Indeed, if a number is a special kind of information, this statement very bold at that time, was not only justified, but also confirmed by the modern practice of its use on computers where three well-known methods of representing data are implemented: numerical (or digitized), symbolic (or textual) and analog (images, sound, and video). All three methods exist simultaneously.

Pic. 30. Pythagoras



A strikingly bold statement even for our time that thinking is an unconscious process of computations, have been expressed in the 17th century by Gottfried Leibniz. Here, thinking is obviously understood as the process of data processing, which in all cases can be represented as numbers. Then it is clear how computations appear, but understanding of the essence of this process in modern science is so far lacking.[34]

Pic. 31. Gottfried Leibniz

[33] An exact definition the notion of data does not exist unless it includes a description from the explanatory dictionary. From here follows the uncertainty of its derivative notions such as data format, data processing, data operations etc. Such vague terminology generates a formulaic thinking, indicating that the mind does not develop, but becomes dull and by reaching in this mishmash of empty words critical point, it simply ceases to think. In this work, a definition the notion of “data” is given in Pt. 5.3.2. But for this it is necessary to give the most general definition the notion of information, which in its difficulty will be else greater than the definition the notion of number since the number itself is an information. The advances in this matter are so significant that after they will follow a real technological breakthrough with such potential of efficiency, which will be incomparably higher than which was due to the advent of computers.

[34] Computations are not only actions with numbers, but also the application of methods to achieve the final result. Even a machine can cope with actions if the mind equips it with appropriate methods. But if the mind itself becomes like a machine i.e. not aware the methods of calculation, then it is able to create only monsters that will destroy also him selves. Namely to that all is going now because of the complete lack of a solution to the problem of ensuring data security. But the whole problem is that informatics as a science simply does not exist.

All definitions of a number have one common basis:

Numbers exist objectively in the sense that they are present in the laws of the world around us, which can be known only through numbers.

From the school bench everyone will learn about numbers from the childish counting: one, two, three, four, five etc. Only the Lord knows where did this counting come from. However, there were attempts to explain its origin using axioms, but the origin of them is as incomprehensible as the counting. Rather, it looks like a certain imitation of the Euclid's "Elements" to add to knowledge the image of science and the appearance of solidity and fundamentality.

The situation is completely different when there is a mathematical definition the essence of a number. Then for a more complete understanding of it, both axioms and a countable quantity become a necessity. Indeed, this definition to the essence of a number includes arguments, actions and a countable quantity. But arguments are also numbers and they should be presented not specifically each of them, but by default i.e. in the form of a generally accepted and unchanged function, which is called the number system, however it no way could to appear without such a notion as a count. Now, axioms turn out to be very appositely and without them a count may be got only from aliens. In reality it was namely so happened since such sources of knowledge as the Euclid's “Elements” or the Diophantus' “Arithmetic” were clearly created not by our, but by a completely different civilization.[35]

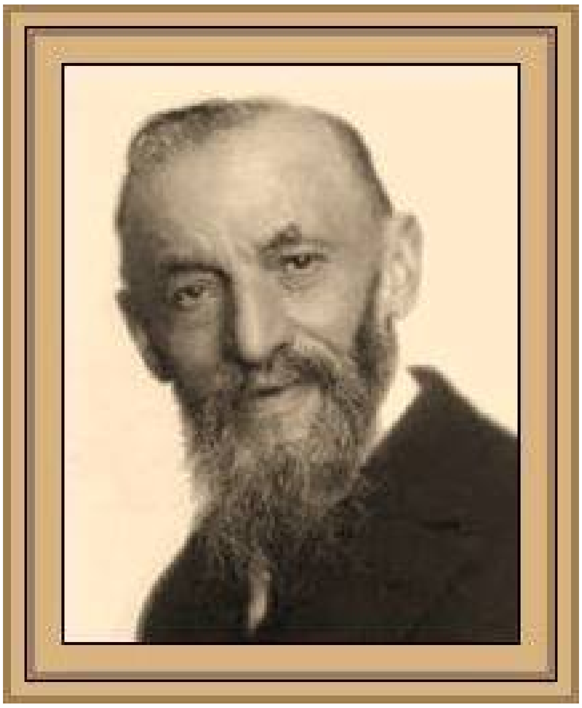
If axioms regulate the count, then they are primary in relation to it. However, there is no need to determine their essence through the introduction of new notions because the meaning of any axioms is precisely in their primacy i.e., they are always essentially *the boundaries of knowledge.* Thus, axioms receive an even more fundamental status, than until now when they were limited only to the foundation of any separate system. In particular, the system of axioms, developed by the Italian mathematician Giuseppe Peano, very closely correspond to the solution of the problem for constructing a counting system although this main purpose was not explained apparently with a hint on justification the essence the notion of number. The scientific community perceived them only as a

[35] Specialists who comment on the ancients in their opinion the Euclid's "Elements" and the Diophantus' "Arithmetic", as if spellbound, see but cannot acknowledge the obvious. Neither Euclid nor Diophantus can be the creators the content of these books, this is beyond the power of even modern science. Moreover, these books appeared only in the late Middle Ages when the necessary writing was already developed. The authors of these books were just translators of truly ancient sources belonging to another civilization. Nowadays, people with such abilities are called medium.

kind of "formalization of arithmetic" completely not noticing that these axioms in no way reflect the essence of numbers, but only create the basis for their presentation by default i.e. through a count.

If the main content of axioms is to determine the boundaries of knowledge related to generally accepted methods of representing of numbers, then they should be built both from the definition the essence of the notion of number and in order to ensure the strength and stability of the whole science's building. Until now, due to the lack of such an understanding of the ways of building the foundations of knowledge, the question about the essence of numbers has never even been asked, but only complicated and confused.

Pic. 32. Giuseppe Peano



However, now when it becomes clearer and without any special difficulties, all science can receive a new and very powerful impetus for its development. And then namely on such a solid basis, science acquires the ability to overcome with an incredible ease such complex obstacles, which in the old days, when there was no understanding the essence of numbers, they seemed to science as completely impregnable fortresses[36].

[36] If from the very beginning we have not decided on the concept of a number and have an idea of it only through prototypes (the number of fingers, or days of the week etc.), then sooner or later we will find that we don't know anything about numbers and follow the calculations an immense set of empirical methods and rules. However, if initially we have an exact definition the notion of number, then for any calculations, we can use only this definition and the relatively small list of rules following from it. If we ourselves creating the required numbers, we can do this through the function arguments, which are represented in the generally accepted number system. But when it is necessary to calculate unknown numbers corresponding to a given function and task conditions, then special methods will often be required, which without understanding the essence of numbers will be very difficult.

3.2. Axioms of Arithmetic

3.2.1. Axioms of a Count

This path was first paved at the end of the 19th century by Peano axioms.[37] We will make changes to them based on our understanding the essence of the number.

Axiom 1*. A number is natural if it is added of units.*[38]

Axiom 2*. The unit is the initial natural number.*

Axiom 3*. All natural numbers form an infinite row, in which each following number is formed by adding unit to the previous number.*

Axiom 4*. The unit does not follow any natural number.*

Axiom 5*. If some proposition is proven for unit (the beginning of induction) and if from the assumption that it is true for a natural number N, it follows that it is also true for a natural number following N (induction hypothesis), then this sentence will be true for all natural numbers.*

Axiom 6*. In addition to natural numbers, there can exist another numbers derived from them, but only in the case if they possess all without exception the basic properties of natural numbers.*

The first axiom is a direct consequence from definition the essence of number, so Peano simply could not have it. Now this first axiom conveys the meaning of defining the notion of number to all another axiom. The second, fourth, and fifth axioms are preserved as in Peano version almost unchanged, but the fourth axiom of Peano is completely removed from this new system as redundant.

The second axiom has the same meaning as the first one in the Peano list, but is being specified in order to become a consequence of the new first axiom.

The third axiom is the new wording of Peano's second axiom. The notion of the natural row is given here more simply than by Peano where you need to guess about it through the notion of the "next" number. The fourth axiom is exactly the same as the third axiom of Peano.

The fifth axiom is the same as by Peano, which is considered the main result of the entire system. In fact, this axiom is the formulation the method of induction, which is very valuable for science and in this case allows to justify and build a count system. However, a count is present in one or another form not only in natural numbers, but also in any other numbers, therefore one more final axiom is needed.

The sixth axiom extends the basic properties of natural numbers to any numbers derived from them because if it turns out that any quantities obtained by calculations from natural numbers, contradict their basic properties, then these quantities cannot belong to the category of numbers.

Now arithmetic gets all the prerequisites in order to have the status the most fundamental of all scientific disciplines. From the point of view the essence of a count everything becomes much simpler and more understandable than until now. On the basis of this updated system of axioms there is no need to "create" natural numbers one after another and then "prove" the action of addition and

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[37] The content of Peano's axioms is as follows: (A1) 1 is a natural number; (A2) For any natural number n there is a natural number denoted by n' and called the number following n; (A3) If m'=n' for any positive integers m, n then m = n; (A4) The number 1 does not follow any natural number i.e. n' is never equal to 1; (A5) If the number 1 has some property P and for any number n with the property P the next number n' also has the property P then any natural number has the property P.

[38] In the Euclid's "Elements" there is something similar to this axiom: "1. An *unit* is that by virtue of each of the things that exist is called one. 2. A *number* is a multitude composed of units" (Book VII, Definitions).

multiplication for the initial numbers. Now it's enough just to give names to these initial numbers within the framework of the generally accepted number system.

If this system is decimal, then the symbols from 0 to 9 should receive the status of the initial numbers composed of units in particular: the number "one" is denoted as 1=1, the number "two" is denoted as 2=1+1, the number " three " as 3=1+1+1 etc. up to the number nine. Numbers after 9 and up to 99 adding up from tens and ones for example, 23=(10+10)+(1+1+1) and get the corresponding names: ten, eleven, twelve … ninety-nine. Numbers after 99 are made up of hundreds, tens and units, etc. Thus, the names of only the initial numbers must be preliminarily counted from units. All other numbers are named so that their quantity can be counted using only the initial numbers.[39]

3.2.2. Axioms of Actions

All arithmetic actions are components of the definition the essence of the number. In a compact form they are presented as follows:

1. Addition: n=(1+1…)+(1+1+1…)=(1+1+1+1+1…)
2. Multiplication: a+a+a+…+a=a×b=c
3. Exponentiation: a×a×a×…×a=a^b=c
4. Subtraction: a+b=c → b=c−a
5. Division: a×b=c → b=c:a
6. Logarithm: $a^b=c \rightarrow b=\log_a c$

Hence, necessary definitions can be formulated in the form of axioms.

Axiom 1. *The action of adding several numbers (summands) is their association into one number (sum).*

Axiom 2. *All arithmetic actions are either addition or derived from addition.*

Axiom 3. *There are direct and inverse arithmetic actions.*

Axiom 4. *Direct actions are varieties of addition. Besides the addition itself, to them also relate multiplication and exponentiation.*

Axiom 5. *Inverse actions are the calculation of function arguments. These include subtraction, division and logarithm.*

Axiom 6. *There aren't any other actions with numbers except for combinations of six arithmetic actions.*[40]

3.2.3. Basic Properties of Numbers

The consequence to the axioms of actions are the following basic properties of numbers due to the need for practical calculations:

1. Filling: a+1>a
2. The neutrality of the unit: a×1=a:1=a

[39] So, count is the nominate starting numbers in a finished (counted) form so that on their basis it becomes possible using a similar method to name any other numbers. All this of course, is not at all difficult, but why is it not taught at school and simply forced to memorize everything without explanation? The answer is very simple – because science simply does not know what a number is, but in any way cannot acknowledge this.

[40] The axioms of actions were not separately singled out and are a direct consequence of determining the essence a notion of number. They contribute both to learning and establish a certain responsibility for the validity of any scientific research in the field of numbers. In this sense, the last 6th axiom looks even too categorical. But without this kind of restriction any gibberish can be dragged into the knowledge system and then called it a "breakthrough in science".

3. Commutativity: a+b=b+a; ab=ba
4. Associativity: (a+b)+c=a+(b+c); (ab)c=a(bc)
5. Distributivity: (a+b)c=ac+bc
6. Conjugation: a=c → a±b=b±c; ab=bc; a:b=c:b; ab=cb; $\log_b a = \log_b c$

These properties have long been known as the basics of primary school and so far, they have been perceived as elementary and obvious. The lack of a proper understanding of the origin of these properties from the essence the notion of number has led to the destruction of science as a holistic system of knowledge, which must now be rebuilt beginning from the basics and preserving herewith everything valuable that remains from real science.

The presented above axiomatics proceeds from the definition the essence the notion of number and therefore represents a single whole. However, this is not enough to protect science from another misfortune i.e. so that in the process of development it does not drown in the ocean of its own researches or does not get entangled in the complex interweaving of a great plurality of different ideas.

In this sense, it must be very clearly understood that axioms are not statements accepted without proof. Unlike theorems, they are only statements and limitations synthesized from the experience of computing, without of which they simply cannot be dispensed. Another meaning is in the basic theorems, which are close to axioms, but provable. One of them is the Basic or Fundamental theorem of arithmetic. This is such an important theorem that its proof must be as reliable as possible, otherwise the consequences may be unpredictable.

Pic. 33. Initial Numbers Pyramids

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$
$$12345 \times 9 + 6 = 111111$$
$$123456 \times 9 + 7 = 1111111$$
$$1234567 \times 9 + 8 = 11111111$$
$$12345678 \times 9 + 9 = 111111111$$
$$123456789 \times 9 + 10 = 1111111111$$

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

$$1 \times 1 = 1$$
$$11 \times 11 = 121$$
$$111 \times 111 = 12321$$
$$1111 \times 1111 = 1234321$$
$$11111 \times 11111 = 123454321$$
$$111111 \times 111111 = 12345654321$$
$$1111111 \times 1111111 = 1234567654321$$
$$11111111 \times 11111111 = 123456787654321$$
$$111111111 \times 111111111 = 12345678987654321$$

3.3. The Basic Theorem of Arithmetic

3.3.1. Mistakes of the Greats and the Fermat's Letter-Testament

The earliest known version of the theorem is given in the Euclid's "Elements" Book IX, Proposition 14.

If a number be the least that is measured by prime numbers, it will not be measured by any other prime number except those originally measuring it.

The explain is following: “Let the number A be the least measured by the prime numbers B, C, D. I say that A will not be measured by any other prime number except B, C, D”. The proof of this theorem looks convincing only at first glance and this visibility of solidity is strengthened by a chain of references: IX-14 → VII-30 → VII-20 → VII-4 → VII-2.

However, an elementary and even very gross mistake was made here. Its essence is as follows:

Let A=BCD where the numbers B, C, D are primes. If we now assume the existence of a prime E different from B, C, D and such that A=EI then we conclude that in this case A=BCD is not divisible by E.

This last statement is not true because the theorem has not yet been proven and it doesn’t exclude for example, BCD=EFGH where E, F, G, H are primes other than B, C, D. Then

A:E=BCD:E=EFGH:E=FGH

i.e. in this case it becomes possible that the number A can be divided by the number E and then the proof of the theorem is based on an argument that has not yet been proven, therefore, the final conclusion is wrong. The same error can take place also in other theorems using decomposition of integers into prime factors. Apparently, due to the archaic vocabulary Euclid's “Elements” even such a great scientist as Euler did not pay due attention to this theorem, otherwise, he would hardly have begun to use “complex numbers” in practice that are not subordinate to it.

The same story happened with Gauss who also did not notice this theorem in the Euclid's "Elements", but nevertheless, formulated it when a need arose. The formulation and proof of Gauss are follows:

“*Each compose number can be decomposed into prime factors in a one only way.*

If we assume that a composite number A equal to $a^{\alpha}b^{\beta}c^{\gamma}$ *…, where a, b, c, … denote different primes, can be decomposed into prime factors in another way, then it is first of all clear that in this second system of factors, there cannot be other primes except a, b, c, …, because the number A composed of these latter cannot be divisible by any other prime number*” [11, 25].

This is an almost exact repeating of erroneous argument in the Euclid's proof. But if this theorem is not proven, then the whole foundation of science built on natural numbers collapses and all the consequences of the definitions and axioms lose their significance. And what to do now? If such giants of science as Euclid and Gauss could not cope with the proof of this theorem, then what we sinners can to do. But yet there is a way out and it is indicated in one amazing document called "Fermat's Letter-Testament".

This letter was sent by Fermat in August 1659 to his longtime friend and former colleague in the Parliament of Toulouse the royal librarian Pierre de Carcavy from whom he was received by the famous French scientist Christian Huygens who was the first to head the French Academy of Sciences created in 1666. Here we give only some excerpts from this Fermat's letter, which are of particular interest to us [9, 36].

“Summary of discoveries in the science about numbers. …

1. Since the usual methods set in the Books are not sufficient to prove very difficult sentences, I finally found a completely special way to solve them. I called this method of proof infinite or indefinite descent. At first, I used it only to prove negative sentences such as: … that there exists no a right triangle in numbers whose area is a square". See Appendix II for details.

The science about numbers is called here arithmetic and the further content of the letter leaves no doubt about it. Namely with arithmetic not only mathematical, but also all other sciences begin. In arithmetic itself the descent method is one of the fundamental one. The following are examples of problems whose solution without this method is not only very difficult, but sometimes even hardly to be possible. Here we will name only a few of these examples.

"*2. For a long time, I could not apply my method to affirmative sentences because rounds and circuitous ways to achieve the aim are much more difficult than those that served me for negative sentences. Therefore, when I needed to prove that every prime number that is by unit more than multiple of four, consists of <sum of> two squares, I was in a greatest difficulty. But finally, my thoughts repeated many times shed light that I did not have and the affirmative sentence became possible to interpret with my method using some new principles that needed to be attached to them. This progress in my reasoning for the case of affirmative sentences is as follows: if some prime number that on 1 exceeds the multiplied of 4, does not consist of two squares, then there is another a prime number of the same nature, smaller than this and then a third, also smaller etc. going down until you come to the number 5, which is the smallest from all numbers of this nature. It therefore, cannot consist of two squares, what however, takes place. From this by proof from the contrary we can conclude that all primes of this nature should consist of two squares*".

This Fermat's theorem was first proven by Euler in 1760 [6, 38], (see Appendix III), and in the framework of the very complex Gauss' "Deductive Arithmetic" this theorem is proving in one sentence [23]. However, no one succeeded in repeating the proof of Fermat himself.

"*… 3. There are infinitely many questions of this kind, but there are others that require new principles for applying the descent method to them … This is the next question that Bachet as he confesses in his commentary on Diophantus, could not prove. On this occasion, Descartes made the same statement in his letters acknowledging that he considers it so difficult that he sees no way to solve it. Each number is a square or consists of two, three or four squares*".

Else earlier, 22 years ago, in October 1636 in a letter to Mersenne Fermat reported on the same problem as about his discovery, but in general form i.e. for any polygonal numbers (for example, triangles, squares, pentagons etc.). Subsequently, he even called this theorem golden one. Consequently, the method of descent was discovered by him at the very beginning of his research on arithmetic. By the time of writing the letter-testament, Fermat already knew from Carcavy that the question of foundation the French Academy of Sciences was practically resolved and he needed only to wait for the building to be completed, so it come true his life's dream to become a professional scientist in the rank of academician. Huygens was commissioned to collect materials for the first academic publications. Fermat proposed for them the method of descent discovered by him and the solution of specific arithmetic tasks on its basis.

However, only few people knew that these tasks were very difficult and Fermat understood that if he would publish their solutions, they would not make any impression at all. He already had such an experience and now he has prepared a real surprise. For those who don't appreciate the value of his solution, he would offer to solve another task. This is the Basic theorem of arithmetic, which is of particular importance for all science since without it the whole theory loses its strength. Fermat found a mistake in the proof of Euclid and came to the conclusion that to prove this theorem without applying the descent method is extremely difficult if at all possible. However, now we can also reveal this secret with the help of our opportunities to look into Fermat's cache with "heretical writings" and return his lost proof to science in the form of the reconstruction presented below.

3.3.2. The Proof of Fermat

So, to prove the Basic theorem of arithmetic we suppose that there exist equal natural numbers A, B consisting of different prime factors:

$A=B$ (1)

where $A=pp_1p_2\ldots p_n$; $B=xx_1x_2\ldots x_m$; $n\geq 1$; $m\geq 1$

Due to the equality of the numbers A, B each of them is divided into any of the prime numbers p_i or x_i. Each of the numbers A, B can consist of any set of prime factors including the same ones, but at the same time there is no one p_i equal to x_i among them, otherwise they would be in (1) reduced. Now (1) can be represented as:

$pQ=xY$ (2)

where p, x are the minimal primes among p_i, x_i; $Q=A/p$; $Y=B/x$.

Since the factors p and x are different, we agree that $p>x$; $x=p-\delta_1$ then

$pQ=(p-\delta_1)(Q+\delta_2)$ (3)

where $\delta_1=p-x$; $\delta_2=Y-Q$

From (3) it follows that $Q\delta_1=(p-\delta_1)\delta_2$ or

$Q\delta_1=x\delta_2$ (4)

Equation (4) is a direct consequence of assumption (1). The right side of this equation explicitly contains the prime factor x. However, on the left side of equation (4) the number δ_1 cannot contain the factor x because $\delta_1 = p - x$ is not divisible by x due to p is a prime. The number Q also does not contain the factor x because by our assumption it consists of factors p_i among which there is not a single equal to x. Thus, there is a factor x on the right in equation (4), but not on the left. Nevertheless, there is no reason to argue that this is impossible because we initially assume the existence of equal numbers with different prime factors.

Then it remains only to admit that if there exist natural numbers A = B composed of different prime factors, then it is necessary that in this case there exist another natural number $A_1=Q\delta_1$ and $B_1=x\delta_2$; also equal to each other and made up of different prime factors. Given that $\delta_1=(p-x)<p$, and $\delta_2=(Y-Q)<Y$ and also, after comparing equation (4) with equation (2), we can state:

$A_1 = B_1$, where $A_1<A$; $B_1<B$ (5)

Now we get a situation similar to the one with numbers A, B only with smaller numbers A_1, B_1. Analyzing now (5) in the manner described above we will be forced to admit that there must exist numbers

$A_2=B_2$, where $A_2<A_1$; $B_2<B1$ (6)

Following this path, we will inevitably come to the case when the existence of numbers $A_k=B_k$, where $A_k<A_{k-1}$; $B_k<B_{k-1}$ as a direct consequence of assumption (1) will become impossible. Therefore, our initial assumption (1) is also impossible and thus the theorem is proven.[41]

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[41] The reconstructed proof of Fermat excludes the mistake made by Euclid. However, beginning from Gauss, other well-known proofs the Basic theorem of arithmetic repeat this same mistake. An exception is the proof received by the German mathematician

Looking at this very simple and even elementary proof by the descent method naturally a puzzling question arise, how could it happen that for many centuries science not only had not received this proof, but was completely ignorant that it had not any one in general? On the other hand, even being mistaken in this matter i.e. assuming that this theorem was proven by Euclid, how could science ignore it by using the "complex numbers" and thereby dooming itself to destruction from within? And finally, how can one explain that this very simple in essence theorem, on which the all science holds, is not taught at all in a secondary school?

As for the descent method, this proof is one of the simplest examples of its application, which is quite rare due to the wide universality of this method. More often, the application of the descent method requires a great strain of thought to bring a logical chain of reasoning under it. From this point of view, some other special examples of solving problems by this method can be instructive.

Ernst Zermelo, see Appendix I.

3.4. The Descent Method

3.4.1. A Little Bit of " Sharpness of Mind" for a Very Difficult Task

We will now consider another example of the problem from Fermat's letter-testament, which is formulated there as follows:

There is only one integer square, which increased by two, gives a cube, this square is 25.

When at the suggestion of Fermat, the best English mathematician of the time John Wallis tried to solve it, he was very vexed and forced to acknowledge he could not do it. For more than two centuries it was believed that Leonard Euler received the solution to this problem, but his proof is based on the use of "complex numbers", while we know these are not numbers at all because they do not obey the Basic theorem of arithmetic. And only at the end of the twentieth century André Weil using the Fermat's triangles method still managed to get a proof [17].

It was a big progress because a purely arithmetic method was used here, however, as applied to this problem, it was clearly dragged the ears. Could Fermat solve this problem easier? We will also extract the answer to this question from the cache, what will allow us to reveal this secret of science in the form of the following reconstruction. So, we have the equation $p^3=q^2+2$ with the obvious solution $p=3$, $q=5$. To prove Fermat's assertion, we suppose that there is another solution $P>p=3$, $Q>q=5$, which satisfies the equation

$P^3=Q^2+2$ (1)

Since it is obvious that $Q>P$ then let $Q=P+\delta$ (2)

Substituting (2) in (1) we obtain: $P^2(P–1)–2\delta P–\delta^2=2$ (3)

Here we need just a little bit of “sharpness of mind” to notice that $\delta>P$ otherwise equation (3) is impossible. Indeed, if we make a try $\delta=P$ then on the left (3) there will be $P^2(P–4)>2$ what is not suitable, therefore there must exist a number $\delta_1=\delta–P$. Then substituting $\delta=P+\delta_1$ in (3) we obtain

$P_2(P–4)–4\delta_1P–\delta_1{}^2=2$ (4)

Now we will certainly notice that $\delta_1>P$ otherwise, by the same logic as above, on the left (4) we get $P^2(P–9)>2$ what again does not suitable, then there must exist a number $\delta_2=\delta_1–P$ and after substituting $\delta_1=P+\delta_2$ in (4), we obtain $P^2(P–9)–6\delta_2P–\delta_2{}^2=2$ (5)

Here one can no longer doubt that this will continue without end. Indeed, by trying $\delta_i=P$ each time we get $P^2(P-K_i)>2$. Whatever the number of K_i this equation is impossible because if $K_i<P$ and $P>3$ then $P^2(P-K_i)>2$ and if $K_i\geq P$ then this option is excluded because then $P^2(P-K_i)\leq 0$

To continue so infinitely is clearly pointless, therefore our initial assumption of the existence of another solutions $P>3$, $Q>5$ is false and this Fermat's theorem is proven.

In the book of Singh, which we often mention, this task is given as an example of the “puzzles” that Fermat was “inventing”. But now it turns out that the universal descent method and a simple technique with trying, make this task one of the very effective examples for learning at school.

Along with this proof, students can easily prove yet another theorem from Fermat’s letter-testament, which could be solved only by such a world-famous scientist as Leonard Euler:

There are only two squares that increased by 4, give cubes, these squares will be 4 and 121.

In other words, the equation $p^3=q^2+4$ has only two integer solutions.

3.4.2. The Fermat's Golden Theorem

We remind that in the Fermat's letter-testament only a special case of this theorem for squares is stated. But also, this simplified version of the task was beyond the power not only of representatives of the highest aristocracy Bachet and Descartes, but even the royal-imperial mathematician Euler.

However, another royal mathematician Lagrange, thanks to the identity found by Euler, still managed to cope with the squares and his proof of only one particular case of FGT is still replicated in almost all textbooks. However, there is no reasonable explanation that the general proof of the FGT for all polygonal numbers obtained by Cauchy in 1815 was simply ignored by the scientific community.

We begin our study with the formulation of the FGT from Fermat's letter to Mersenne in 1636. It is presented there as follows:

Every <natural> number is equal
one, two or three triangles,
one, 2, 3 or 4 squares,
one, 2, 3, 4 or 5 pentagons,
one, 2, 3, 4, 5 or 6 hexagons,
one, 2, 3, 4, 5, 6 or 7 heptagons,
and so on to infinity [36].

Since polygonal numbers are clearly not respected by today's science, we will give here all the necessary explanations. The formula for calculating any polygonal number is represented as

$m_i = i+(k-2)(i-1)i/2$

where m is a polygonal number, i is a serial number, k is the quantity of angles.

Thus, $m_1=1$; $m_2=k$; and for all other i the meaning of mi varies widely as shown in the following table:

Table 1. Polygonal numbers

i	1	2	3	4	5	6	7	8	9
k=3	1	3	6	10	15	21	28	36	45
k=4	1	4	9	16	25	36	49	64	81
k=5	1	5	12	22	35	51	70	92	117
k=6	1	6	15	28	45	66	91	120	153
k=7	1	7	18	34	55	81	112	148	189
k=8	1	8	21	40	65	96	133	176	225
k=9	1	9	24	46	75	111	154	204	261
k	1	k	3k−i	6k−2i	10k−3i	15k−4i	21k−5i	28k−6i	36k−7i

To calculate m_i it is enough to obtain only triangular numbers by the formula, which is very easily since the difference between them grows by unit with each step. And all other m_i can be calculated by adding the previous triangular number in the columns. For example, in column i=2, numbers increase by one, in column i=3 – by three, in column i=4 – by six etc. i.e. just on the value of the triangular number from the previous column.

To make sure that any natural number is represented by the sum of no more than k k-angle numbers is quite easily. For example, the triangular number 10 consists of one summand. Further 11=10+1, 12=6+6, 13=10+3 of two, 14=10+3+1 of three, 15 again of one summand. And so, it will happen regularly with all natural numbers. Surprisingly that the number of necessary summands is limited precisely by the number k. So, what is this miraculous power that invariably gives such a result?

As an example, we take a natural number 41. If as the summand triangular number will be closest to it 36, then it will not in any way to fit into three polygonal numbers since it consists minimum of 4 ones i.e. 41=36+3+1+1. However, if instead of 36 we take other triangular numbers for example, 41=28+10+3, or 41=21+10+10 then again in some unknown miraculous way everything will so as it stated in the FGT.

At first glance it seems simply unbelievable that it can somehow be explained? But we still pay attention to the existence of specific natural numbers, which are consisting at least of k k-angle numbers and denoted by us as S-numbers. Such numbers are easily to find for example, for triangles – 5, 8, 14, for squares – 7, 15, 23, for pentagons – 9, 16, 31 etc. And this our simple observation allows us directly to move to aim i.e. without using ingenious tricks or powerful "sharpness of mind".

Now to prove the FGT, suppose the opposite i.e. that there exists a certain minimal positive integer N consisting minimum of k + 1 k-angle numbers. Then it's clear that this our supposed number should be between some k-angle numbers m_i and m_{i+1} and can be represented as

$N=m_i+\delta_1$ where $\delta_1=N-m_i$ (1)

It is quite obvious that δ_1 must be an S-number since otherwise this would contradict our assumption about the number N. Then we proceed the same way as in our example with the number 41 i.e. represent the supposed number as

$N = m_{i-1}+\delta_2$ where $\delta_2=N-m_{i-1}$

Now δ_2 should also be an S-number. And here so we will go down to the very end i.e. before

$\delta_{i-1}=N-m_2=N-k$ and $\delta_i=N-m_1=N-1$ (2)

Thus, in a sequence of numbers from δ_1 to δ_i, all of them must be S-numbers i.e. each of them will consist of a sum minimum of k k-angle numbers, while our supposed number N will consist minimum of k+1 k-angle numbers. From (1) and (2) it follows:

$N- m_i =S_i$ (3)

Thus, if we subtract any smaller polygonal number m_i from our supposed number N then according to our assumption, the result should be only an S-number. Of course, this condition looks simply unbelievable and it seems that we are already at target, but then how can one prove that this is impossible? …

If we gave an answer to this question here, then this famous Fermat's theorem would immediately turn into the most common school problem and interest to it would be lost. To prevent this from happening, we will stay on the fact that the proof is presented here only by 99% and the remaining 1% will be offered to those who will be interested in order to appreciate the true magnificence of this scientific achievement of Fermat, especially in comparison with the Cauchy's GFT proof.[42]

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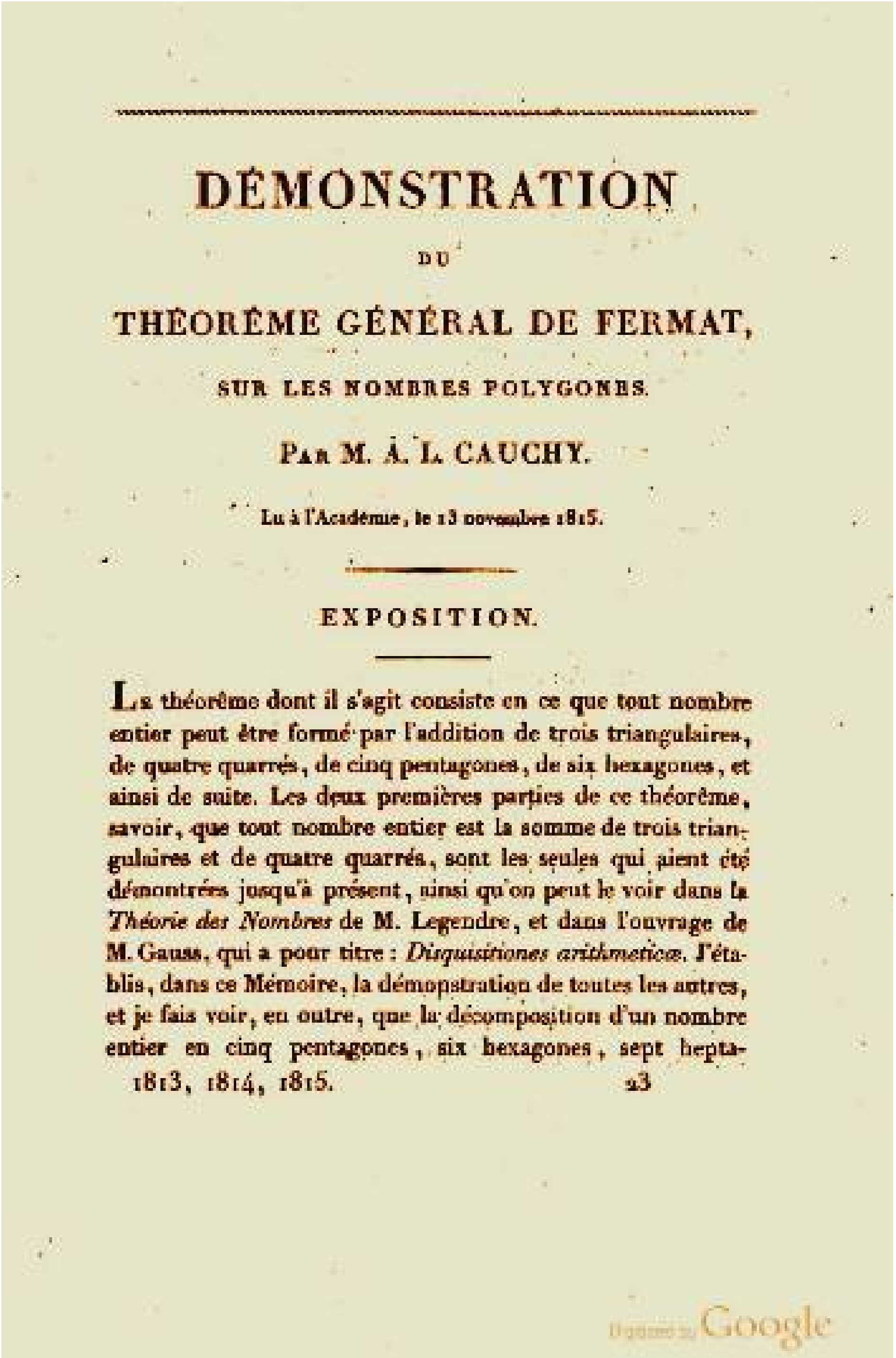
[42] Facsimile of the edition with the Cauchy's GTF proof was published by Google under the title MEMIRES DE LA CLASSE DES SCIENCES MATHTÉMATIQUES ET PHYSIQUES DE L'INSTITUT DE France. ANNEES 1813, 1814, 1815: https://books.google.de/books?id=k2pFAAAAcAAJ&pg=PA177#v=onepage&q&f=false What we need is on page 177 under the title DEMONSTRATION DU THÉORÉME GÉNÉRAL DE FERMAT, SUR LES NOMBRES POLYGONES. Par M. A. L. CAUCHY. Lu à l'Académie, le 13 novembre 1815 (see Pics 34, 35). The general proof of Cauchy takes 43 (!!!) pages and this circumstance alone indicates that it does not fit into any textbook. Such creations are not something that students, but also academics are not be available because the first cannot understand anything about them and the second simply do not have the necessary time for this. Then it turns out that such proofs are hardly possible to check how convincing they are i.e. are they any proofs in general? But if Cauchy applied the descent method recommended by Fermat, then the proof would become so convincing that no checks would be required. A very

Pic. 34. Title Page the Cauchy's Proof
of the *Fermat's Golden Theorem*



DÉMONSTRATION

DU

THÉORÊME GÉNÉRAL DE FERMAT,

SUR LES NOMBRES POLYGONES.

PAR M. A. L. CAUCHY.

Lu à l'Académie, le 13 novembre 1815.

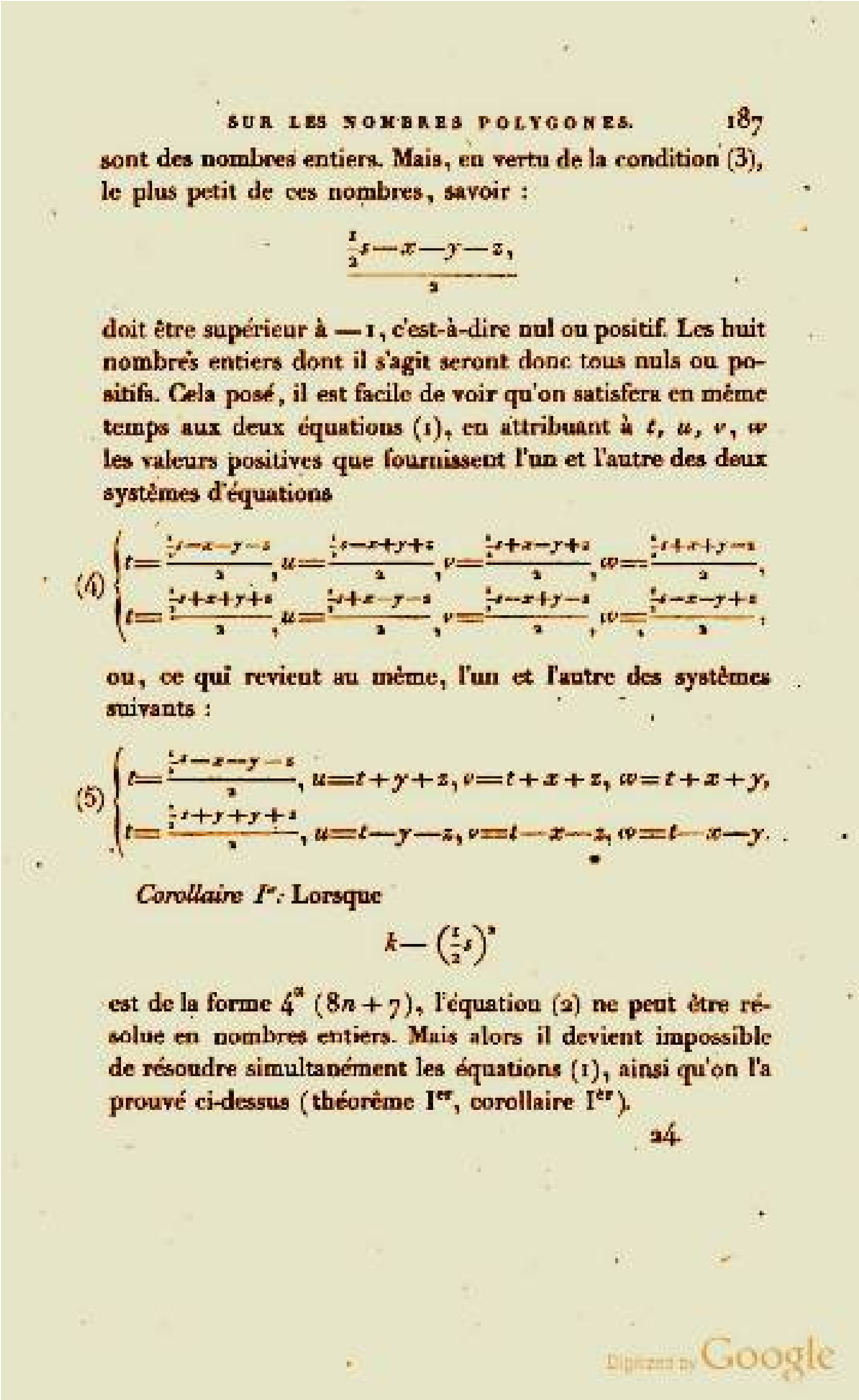
EXPOSITION.

Le théorème dont il s'agit consiste en ce que tout nombre entier peut être formé par l'addition de trois triangulaires, de quatre quarrés, de cinq pentagones, de six hexagones, et ainsi de suite. Les deux premières parties de ce théorème, savoir, que tout nombre entier est la somme de trois triangulaires et de quatre quarrés, sont les seules qui aient été démontrées jusqu'à présent, ainsi qu'on peut le voir dans la *Théorie des Nombres* de M. Legendre, et dans l'ouvrage de M. Gauss, qui a pour titre : *Disquisitiones arithmeticæ*. J'établis, dans ce Mémoire, la démonstration de toutes les autres, et je fais voir, en outre, que la décomposition d'un nombre entier en cinq pentagones, six hexagones, sept hepta-

1813, 1814, 1815. 23

Pic. 35. One of 43 Pages the Cauchy's Proof
of the *Fermat's Golden Theorem*

simple conclusion follows from this: *The Fermat's Golden Theorem* as well as some of his other theorems, still remain unproven.



SUR LES NOMBRES POLYGONES. 187

sont des nombres entiers. Mais, en vertu de la condition (3), le plus petit de ces nombres, savoir :

$$\frac{\frac{1}{2}s-x-y-z}{2},$$

doit être supérieur à -1, c'est-à-dire nul ou positif. Les huit nombres entiers dont il s'agit seront donc tous nuls ou positifs. Cela posé, il est facile de voir qu'on satisfera en même temps aux deux équations (1), en attribuant à t, u, v, w les valeurs positives que fournissent l'un et l'autre des deux systèmes d'équations

$$(4)\begin{cases} t=\dfrac{\frac{1}{2}s-x-y-z}{2},\ u=\dfrac{\frac{1}{2}s-x+y+z}{2},\ v=\dfrac{\frac{1}{2}s+x-y+z}{2},\ w=\dfrac{\frac{1}{2}s+x+y-z}{2}, \\ t=\dfrac{\frac{1}{2}s+x+y+z}{2},\ u=\dfrac{\frac{1}{2}s+x-y-z}{2},\ v=\dfrac{\frac{1}{2}s-x+y-z}{2},\ w=\dfrac{\frac{1}{2}s-x-y+z}{2}; \end{cases}$$

ou, ce qui revient au même, l'un et l'autre des systèmes suivants :

$$(5)\begin{cases} t=\dfrac{\frac{1}{2}s-x-y-z}{2},\ u=t+y+z,\ v=t+x+z,\ w=t+x+y, \\ t=\dfrac{\frac{1}{2}s+x+y+z}{2},\ u=t-y-z,\ v=t-x-z,\ w=t-x-y. \end{cases}$$

Corollaire Ier: Lorsque

$$k-\left(\frac{1}{2}s\right)^2$$

est de la forme $4^a\,(8n+7)$, l'équation (2) ne peut être résolue en nombres entiers. Mais alors il devient impossible de résoudre simultanément les équations (1), ainsi qu'on l'a prouvé ci-dessus (théorème Ier, corollaire Ier).

24

3.4.3. Archimedes-Fermat Problem

The problem statement is as follows:

Let any non-square number be given, you need to find an infinite number of squares, which after multiplication by this number and increasing by unit, will make a square.

Fermat proposed finding solutions for the numbers 61, 109, 149, and 433 [36].

The English mathematician John Wallis managed to find a way to calculate the required numbers using the Euclidean method of decomposing an irrational number into an infinite common fraction. He published his decision under the name "Commercium epistolicum" see pic. 37-38.

Pic. 36. John Wallis

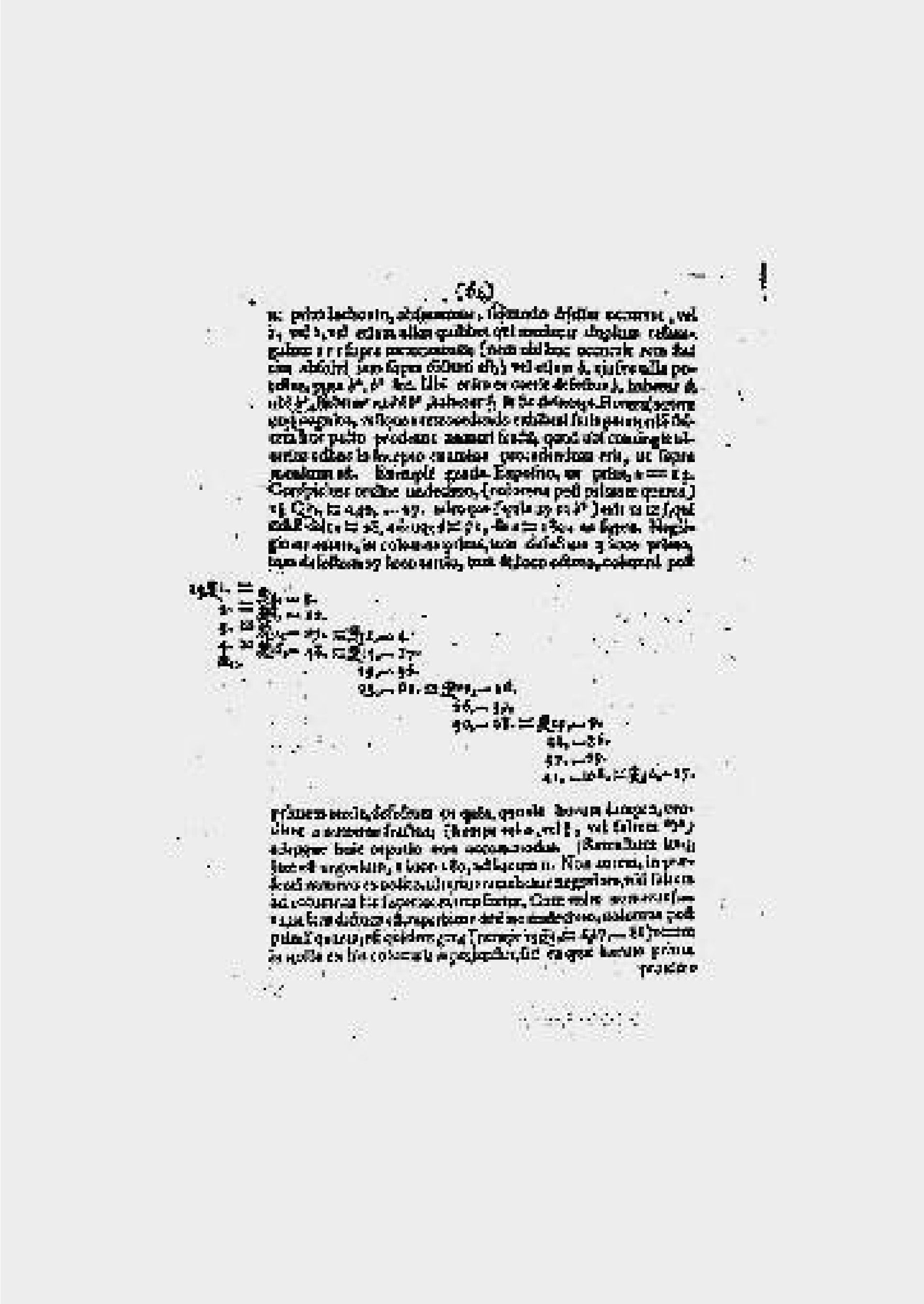
Pic. 37. Title Page of Wallis's Publication *Commercium Epistolicum*


COMMERCIUM
EPISTOLICUM.

Although Wallis did not give a complete proof the validity of this method, Fermat nevertheless admitted that he had coped with the task. Euler came very close to the solution when he showed that this fraction is cyclical, but he was not able to complete the proof and this task was finally solved by Lagrange. Later, this Fermat's task also was solved by Gauss in his own way, but for this purpose the extensive theory he created called “Arithmetic of deductions” was involved. And everything would be fine if the Lagrange's proof was not in the category of highest difficulty and the Gauss decision was not based on the most complicated theory. Fermat himself clearly could not follow such ways. About how he himself solved this problem, he reports in the letter-testament to Carcavy in August 1659 [36]: “I recognize that Mr. Frenicle gave various special solutions to this question as well as Mr. Wallis, but a common solution will be found using the method of descent applied skillfully and appropriately." However, this Fermat's solutions so remained as the secret behind seven seals!

Pic. 38. Page 64 *Commercium Epistolicum*

Demonstrating Wallis Method



We will try here slightly to open the veil over this mystery. To do this, we will look at a simple example of Wallis calculations and then compare it with how one could do these calculations using Fermat's method. So, we need to find the smallest numbers x and y that satisfy the equation $Ax^2 + 1 = y^2$. Let A = 29 then calculations by the Wallis method look as follows [32]:

$a=\sqrt{29}$	$a_0= \|a\| =5$
$a_1=1/(a_0-5)=1/(\sqrt{29}-5)=(\sqrt{29}+5)/4$	$a_1=\|a_1\|=2$
$a_2=1/(a_1-2)=1/(\sqrt{29}-5)=(\sqrt{29}+3)/5$	$a_2=\|a_2\|=1$
$a_3=1/(a_2-1)=1/(\sqrt{29}-2)=(\sqrt{29}+2)/5$	$a_3=\|a_3\|=1$
$a_4=1/(a_3-1)=1/(\sqrt{29}-3)=(\sqrt{29}+3)/4$	$a_4=\|a_4\|=2$
$a_5=1/(a_5-2)=4)=(\sqrt{29}+5)$	$a_5=\|a_5\|=10$
$a_6=1/(\sqrt{29}-5)= a_1$	

From this sequence of calculations, a chain of suitable fractions is obtained by backward i.e. from a5 to a0 and looks like: 5/1; 11/2; 16/3; 27/5. As a result, we get 70/13. Then the minimum solution would be:

$x_1\sqrt{29}+y_1=(13\sqrt{29}+70)^2=1820\sqrt{29}+9801$; $x_1=1820$; $y_1=9820$

Wallis was unable to prove that this method of computation gives solutions for any non-square number A. However, he guessed that the chain of computations ends where a6 will be computed by the same formula as a1. To understand the meaning of this chain of calculations, you need to study a very voluminous and extremely difficult theory [7, 14, 19, 23, 26, 32], which Fermat could not have developed at that time. Since no Fermat's manuscripts on arithmetic have survived, a natural question arises: how could he formulate such a difficult problem, about which there was very little information before him?

For today's science such a question is clearly beyond its capabilities since for it the pinnacle of achievements in solving Fermat's problems is any result even inflated to such incredible dimensions that we have today. However, it is difficult to imagine how much this our respected science will be dejected when from this book it learns that the problem was solved by Fermat not for great scientists, but … for schoolchildren!!! However, here we cannot afford to grieve science so much, so we only note that the example given in the textbooks is very unfortunate since it can be solved quite simply, namely: $x = 2mz$, where $m<x$, $z<y$, $Am^2-1 = z^2$. This last equation differs from the initial one only in sign and even by the method of ordinary tests without resorting to irrational numbers one can easily find the solution $m = 13$; $z = 70$; $x = 2 \times 13 \times 70 = 1820$; $y = 9820$.

Obviously, in textbooks it would be much more appropriate to demonstrate an example with the number 61 i.e. the smallest number proposed by Fermat himself. How he himself solved this problem is unknown to science, but we have already repeatedly demonstrated that it is not a problem for us to find out. We just need to look once more into the cache of the Toulousean senator and as soon as we succeeded, we quickly found the right example so that it could be compared with the Wallis method. In this example you can calculate $x = 2mz$, where m and z are solutions to the corresponding equation $61m^2 - z^2 = 1$. Then the chain of calculations is obtained as follows:

$61m^2-z^2=1$

$m=(8m_1\pm z_1)/3=(8\times722+5639)/3=3805$; $z^2=61\times3805^2-1=29718^2$

$61m_1^2-z_1^2=3$

$m=(8m_1\pm z_1)/3=(8\times722+5639)/3=3805$; $z_1^2=61\times722^2-1=29718^2$

$61m_2^2-z_2^2=9$

$m=(8m_1\pm z_1)/3=(8\times722+5639)/3=3805$; $z_2^2=61\times137^2-1=29718^2$

$61m_3^2-z_3^2=27$

$m_3=(8m_4\pm z_4)/3=(8\times5+38)/3=26;\ z_3^2=61\times26^2-27=203^2$

$61m_4^2-z_4^2=81$

$m_4=(8m_5\pm z_5)/3=(8\times2-1)/3=5;\ z_4^2=61\times5^2-81=38^2$

$61m_5^2-z_5^2=243$

$m_5=2;\ z_5^2=1$

We will not reveal all nuances of this method, otherwise all interest to this problem would have been lost. We only note that in comparison with Wallis method where the descent method is not used, here it is present in an explicit form. This is expressed in the fact that if the numbers m and z satisfying the equation $61m^2-z^2=1$ exist, then there must still exist numbers $m_1<m$ and $z_1<z$ satisfying the equation $61m_1^2-z_1^2=3$, as well as the numbers $m_2<m_1$ and $z_2<z_1$, from equation $61m_2^2-z_2^2=9$, etc. up to the minimum values $m_5<m_4$ and $z_5<z_4$. The number 3 appearing in the descent is calculated as 64 – 61, that is, as the difference between 61 and the square closest to it. Calculations as well as in the Wallis method are carried out in the reverse order i.e. only after the minimum values of m_5 and z_5 have been calculated. As a result, we get:

$m=3805;\ z=29718$

$x=2mz=2\times3805\times29718=226153980$

$y=\sqrt{(61\times2261539802+1)}=1766319049$

Of course, connoisseurs of the current theory will quickly notice in this example that the results of calculations obtained in it will exactly coincide with those that can be obtained by the Wallis' method. However, for this they will have to use the irrational number √61, and our example with Fermat's method showed that it is possible to do calculations exclusively in the framework of arithmetic i.e. only in natural numbers. There is no doubt also that experts without much effort will guess how to get the formulas shown in our example. However, it will not be easily for them to explain how to apply this Fermat's method in the general case because from our example it is not at all clear how it is possible to determine that the ultimate goal is to solve the equation $61m_5^2 - z_5^2 = 243$ from which calculations should be performed with a countdown.

It would be simply excellent if today's science could explain Fermat's method in every detail, but even the ghostly hopes for this are not yet visible. It would be more realistic to expect that attempts will be made to refute this example as demonstration a method of solving the problem unknown to science. Nevertheless, science will have to reckon with the fact that this example is still the only one in history (!!!) confirmation of what Fermat said in his letter-testament. When this secret is fully revealed, then all skeptics will be put to shame and they will have no choice, but to recognize Fermat as greater than all the other greatest scientists because they were recognized as such mainly because they created theories so difficult for normal people to understand that they could only cause immense horror among students who now have to take the rap for such a science.[43]

https://www.youtube.com/watch?v=wFz8W2HsjfQ
https://www.youtube.com/watch?v=cUytn2SZ1n4
https://www.youtube.com/watch?v=ZhVNOgaBStY

In this sense, the following example of solving a problem using the descent method will be particularly interesting because it was proposed in a letter from Fermat to Mersenne at the end of 1636, i.e. the age of this task is almost four centuries! Euler's proof [8] was incorrect due to the use

[43] Examples are in many videos from the Internet. However, these examples in no way detract from the merits of professors who know their job perfectly.

of "complex numbers" in it. However, even the revised version of André Weil in 1983 [17] is too complex for schooling.

3.4.4. Fermat's Problem with Age 385 years

In the original version in 1636 this task was formulated as follows:

Find two square-squares, which sum is equal to a square-square,
or two cubes, which sum is a cube.

This formulation was used by Fermat's opponents as the fact that Fermat had no proof of the FLT and limited himself to only these two special cases. However, the very name "The Fermat's Last Theorem" appeared only after the publication of "Arithmetic" by Diophantus with Fermat's remarks in 1670 i.e. five years after his death. So, there is no any reason to assert that Fermat announced the FLT in 1637.

The first case for the fourth power we have presented in detail in Appendix II. As for the case for the third power, Fermat's own proof method restored by us below, will not leave any chances to the solutions of this problem of Euler and Weil to remain in history of science, since from the point of view of the simplicity and elegance of the author's solution this problem, they will become just unnecessary.

Now then, to prove that there are no two cubes whose sum is a cube, we use the simplest approach based on divisibility of numbers, what means that in the original equation

$a^3+b^3 = c^3$ (1)

the numbers a, b, and c can be considered as coprime ones, i.e. they do not have common factors, but in general case this is not necessary, since if we prove that equation (1) cannot have solutions in any integers, including those with common factors, then we will prove that coprime numbers also cannot be solutions of the original equation. Then we assume that both sides of equation (1) in all cases must be divisible by the number c^2, then equation (1) can be represented as

$c^3 = c^2(x+y) = a^3+b^3$ (2)

In this case, it is easily to see that there is only one way to get solutions to equation (1) when the numbers c, x, y, and x+y are cubes, i.e.

$c = x+y = p^3+q^3= z^3; x = p^3; y = q^3$ (3)

Then equation (1) must have the form:

$(z^3)^3 = (z^2)^3(p^3+q^3)$ (4)

Thus, we found that if there are numbers a, b, and c that satisfy equation (1), then there must be numbers $p<a$, $q<b$, and $z<c$ that satisfy equations (3)

$p^3+q^3= z^3$

If we now apply the same approach to solving this equation, that we applied to solving equation (1), we will get the same equation, only with smaller numbers. However, since it is impossible to infinitely reduce natural numbers, it follows that equation (1) has no solutions in integers.

At first glance, we have received a very simple and quite convincing proof of the Fermat problem by the descent method, which no one has been able to obtain in such a simple way for 385 years, and we can only be happy about it. However, such a conclusion would be too hasty, since this proof is actually incorrect and can be refuted in the most unexpected way.

However, this refutation is so surprising that we will not disclose it here, because it opens the way not only for *the simplest proof of the FLT*, but also automatically allows to reduce it to *a very simple proof of the Beal conjecture*. The disclosure the method of refuting this proof would cause a real commotion in the scientific world, therefore we will include this mystery among our riddles (see Appendix V Pt. 41).

So, we have demonstrated here solving to Fermat's problems (only by descent method!):

1) The proof of the *Basic theorem of arithmetic*.

2) The proof of the Fermat's theorem on the unique solving the equation $p^3 = q^2 + 2$.

3) A way to prove *Fermat's Golden Theorem*.

4) A Fermat's way to solve the Archimedes-Fermat equation $Ax^2 + 1 = y^2$.

5) The proof method of impossibility $a^3+b^3=c^3$ in integers, which opens a way to simplest proofs of the FLT and Beal conjecture.

6) A Fermat's proof his grandiose discovery about primes in the form $4n + 1 = a^2 + b^2$ which we have presented in another style in Appendix IV, story *Year 1680*.

Over the past 350 (!!!) years after the publication of these problems by Fermat, whole existing science could not even dream of such a result!

3.5. Parity Method

Before we embarking on the topic "Fermat's Last Theorem" we note that this problem was not solved by Fermat himself using the descent method, otherwise in his FLT formulation there would be no mention of a "truly amazing proof", which certainly related to other methods. Therefore, to the above examples of the application of the descent method we will add our presentation of two methods unknown to today's science. The most curious of these is the parity method.

3.5.1. Defining Parity as a Number

The Basic theorem of arithmetic implies a simple, but very effective idea of defining parity as a number, which is formulated as follows:

The parity of a given number is the quantity of divisions this number by two without a remainder until the result of the division becomes odd.

Let's introduce the parity symbol with angle brackets. Then the expression ‹x› = y will mean:

the parity of the number x is equal to y. For example, the expression "the parity of the number forty is equal to three" can be represented as: ‹40›= 3. From this definition of parity, it follows:

– parity of an odd number is zero.

– parity of zero is infinitely large.

– any natural number n can be represented as $n = 2^w (2N - 1)$

where N is the base of a natural number, w is its parity.

3.5.2. Parity Law

Based on the above definition the parity, it can be stated that equal numbers have equal parity. In relation to any equation this provision refers to its sides and is absolutely necessary in order for it to have solutions in integers. From here follows the parity law for equations:

Any equation can have solutions in integers if and only if the parities of both its sides are equal.

The mathematical expression for the parity law is $W_L = W_R$ where W_L and W_R are the parities of the left and right sides of the equation respectively. A distinctive feature of the parity law is that the equality of numbers cannot be judged by the equality of their parity, but if their parities are not equal, then this certainly means the inequality of numbers.

3.5.3. Parity Calculation Rules

Parity of a sum or difference two numbers a and b

If ‹a› < ‹b› then ‹a ± b› = ‹a›.

It follows in particular that the sum or difference of an even and an odd number always gives a number with parity zero. If ‹a› = ‹b› = x then either ‹a + b› = x + 1 wherein ‹a – b› > x + 1

or ‹a – b› = x + 1 wherein ‹a + b› > x + 1

These formulas are due to the fact that

‹(a + b) + (a – b)› = ‹2a› = ‹a› + 1

It follows that the sum or difference of two even or two odd numbers gives an even number.

Parity of a sum or difference two power number a^n and b^n

If ‹a› < ‹b› then $‹a^n \pm b^n› = ‹a^n›$. If ‹a› = ‹b› = x then

only for even n:

$‹a^n - b^n› = ‹a - b›+ ‹a + b›+ x(n - 2) + ‹n› - 1$

$‹a^n + b^n› = xn + 1$
only for odd n:
$‹a^n \pm b^n› = ‹a \pm b› + x(n - 1)$
When natural numbers multiplying, their parities are added up
$‹ab› = ‹a› + ‹b›$
When natural numbers dividing, their parities are subtracted
$‹a : b› = ‹a› - ‹b›$
When raising number to the power, its parity is multiplied
$‹a^b› = ‹a› \times b$
When extracting the root in number, its parity is divided
$‹\sqrt[b]{a}› = ‹a› : b$

3.6. Key Formula Method

To solve equations with many unknowns in integers, an approach is often used when one more equation is added to the original equation and the solution to the original is sought in a system of two equations. We call this second equation the key formula. Until now due to its simplicity, this method did not stand out from other methods, however we will show here how effective it is and clearly deserves special attention. First of all, we note an important feature of the method, which is that:

Key formula cannot be other as derived from the original equation.

If this feature of the method is not taken into account i.e. add to the original equation some other one, then in this case, instead of solving the original equation we will get only a result indicating the compatibility of these two equations. In particular, we can obtain not all solutions of the original equation, but only those that are limited by the second equation.

In the case when the second equation is derived from the initial one, the result will be exhaustive i.e. either all solutions or insolvability in integers of the original equation. For example, we take equation $z^3 = x^2 + y^2$. To find all its solutions we proceed from the assumption that a prerequisite (key formula) should be $z = a^2 + b^2$ since the right-hand side of the original equation cannot be obtained otherwise than the product of numbers which are the sum of two squares. This is based on the fact that:

The product of numbers being the sum of two squares in all cases gives a number also consisting the sum of two squares.

The converse is also true: if it is given a composite number being the sum of two squares then it cannot have prime factors that are not the sum of two squares. This is easily to make sure from the identity

$(a^2+b^2)(c^2+d^2)=(ac+bd)^2+(ad-bc)^2=(ac-bd)^2+(ad+bc)^2$

Then from $(a^2+b^2)(a^2+b^2)=(aa+bb)^2+(ab-ba)^2=(a^2-b^2)^2+(ab+ba)^2$ it follows that the square of a number consisting the sum of two squares, gives not two decompositions into the sum of two squares (as it should be in accordance with the identity), but only one, since (ab−ba)2= 0 what is not a natural number, otherwise any square number after adding to it zero could be formally considered the sum of two squares.

However, this is not the case since there are numbers that cannot be the sum of two squares.

As Pierre Fermat has established, such are all numbers containing at least one prime factor of type $4n - 1$. Now from

$a^2-b^2=c$; $ab+ba=2ab=d$; $(a^2+b^2)^2=c^2+d^2$

the final solution follows:

$z^3=(a^2+b^2)^3=(a^2+b^2)(c^2+d^2)=x^2+y^2$

where a, b are any natural numbers and all the rest are calculated as $c=a^2-b^2$; $d=2ab$; $x=ac-bd$; $y=ad+bc$ (or $x=ac+bd$; $y=ad-bc$). Thus, we have established that the original equation $z^3=x^2+y^2$ has an infinite number of solutions in integers and for specific given numbers a, b – two solutions.

It is also clear from this example why one of the Fermat's theorems asserts that:

A prime number in the form 4n+1 and its square can be decomposed into two squares only in one way; its cube and biquadrate only in two; its quadrate-cube and cube-cube only in three etc. to infinity.

4. The Fermat's Last Theorem

4.1. The Thorny Path to Truth

4.1.1. The FLT up to now remains unproven

The scientific world has been at first learned about the FLT after publication in 1670 of "Arithmetic" by Diophantus with Fermat's remarks (see Pic. 3 and Pic. 96 from Appendix VI). And since then i.e. for three and a half centuries, science cannot cope with this task. Moreover, perhaps this is namely why the FLT became the object of unprecedented falsification in the history of mathematics. It is very easily to verify this since the main arguments of the FLT "proof" 1995 are well known and look as follows.

If the FLT were wrong, then there would be exist an elliptical "Frey curve" (???): $y^2=x(x-a^n)(x+b^n)$ where $a^n+b^n=c^n$. But Kenneth Ribet has proven that such a curve cannot be modular. Therefore, it suffices to obtain a proof of the Taniyama – Shimura conjecture, that all elliptic curves must be modular, so that it simultaneously becomes a proof of the FLT. The proof was presented in 1995 by Andrew Wiles who became the first scientist that allegedly has proven the FLT.

However, it turns out that the "Frey curve" and together with it the works of Ribet and Wiles have with the FLT nothing to do at all!!![44] And as regards the "proof" of A. Wiles the conjecture of Taniyama – Shimura, he also himself admitted[45] that one needs much more to learn (naturally, from Wiles) in order to understand all of its nuances, setting forth on 130 pages (!!!) of scientific journal "Annals of Mathematics". Quite naturally that after the appearance of such exotic "proof", scientists cannot come to their senses from such a mockery of science, the Internet is replete with all sorts of refutations,[46] and there is no doubt that any generally accepted proof of the FLT still does not exist.

The special significance of the FLT is that in essence, this is one of the simple cases to addition of power numbers when only the sum of two squares can be a square and for higher powers such addition is impossible. However, according to the Waring-Hilbert theorem, any natural number

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[44] It must be admitted that the method of Frey's proof is basically the same as that of Fermat i.e. it is based on obtaining a solution to the equation $a^n+b^n=c^n$ by combining it into a system with another equation – a key formula, and then solving this system. But if Fermat's key formula a+b=c+2m is derived directly from the initial equation, while at Frey it is just taken from nothing and united to the Fermat equation $a^n+b^n=c^n$ i.e. Frey's curve $y^2=x(x-a^n)(x+b^n)$ is a magical trick that allows to hide the essence of the problem and replace it with some kind of illusion. Even if Frey could prove the absence of integer solutions in his equation then this could in no way lead him to the proof of the FLT. But he did not succeed it also, therefore one "brilliant idea" gave birth to an "even more brilliant idea" about the contradiction of the "Frey curve" to the Taniyama – Simura conjecture. With this approach you can get incredibly great opportunities for manipulating and juggling the desired result, for example, you can "prove" that the equation a+b+c=d as well as the Fermat equation $a^n+b^n=c^n$ in integers cannot be solved if take abc=d as a key formula. However, such "ideas" that obviously indicate the substitution the subject of the proof should not be considered at all, since magicians hope only for the difficulty of directly refuting their trick.

[45] Here is how E. Wiles himself comments on a mistake found in his "proof" in 1993: "Even explaining it to a mathematician would require the mathematician to spend two or three months studying that part of the manuscript in great detail". See Nova Internet Publishing http://www.pbs.org/wgbh/nova/physics/andrew-wiles-fermat.html It turns out that this "proof" understood only by its author, while everyone else needs to learn and learn.

[46] Such debunks are very detailed, but too redundant since the arguments of the main authors of the FLT "proof" by G. Frey and E. Wiles look so ridiculous that otherwise as by the hypnotic influence of the unholy it would impossible to explain why many years after 1995 for some reason none of the recognized pundits so have ever noticed that instead of FLT proof we have got a something completely different.

(including an integer power) can be the sum of the same (or equal to a given) powers[47]. And this a much more complex and no less fundamental theorem was proven much earlier than the FLT.

We also note the fact that the FLT attracts special attention not at all because this task is simple in appearance, but very difficult to solve. There are also much simpler-looking tasks, which are not only not to be solved, but also even nobody really knows how to approach them [48]. The FLT especially differs from other tasks that attempts to find its solution lead to the rapid growth of new ideas, which become impulses for the development of science. However, there was so much heaped up on this path that even in very voluminous studies, all this cannot be systematized and combined.[49]

Great scholars did not attach much importance to building the foundations of science apparently considering such creativity to be a purely formal matter, but centuries-old failures with the FLT proof indicate that they underestimated the significance of such studies. Now when it became clear where such an effective scientific tool as the descent method could come from, as well as other tools based on understanding the essence of number, it becomes clear why Fermat was so clearly superior to other mathematicians in arithmetic, while his opponents have long been in complete bewilderment from this obvious fact.

Here we come to the fact that the main reason for failures in the search for FLT proof lies in the difference between approaches to solving tasks by Fermat and other scientists, as well as in the fact that even modern science has not reached the knowledge that already was used by Fermat in those far times. This situation needs to be corrected because otherwise the FLT so will continue to discredit whole science.

One of the main questions in the studies on the FLT was the question of what method did Fermat use to prove this theorem? Opinions were very different and most often it was assumed that this was the method of descent, but then Fermat himself hardly called it "truly amazing proof." He also could not apply the Kummer method, from which the best result was obtained in proving the FLT proof over the last 170 years. But perhaps he besides the descent method had also other ones? Yes indeed, this is also described in detail in treatise "A New Discovery in the Art of Analysis" by Jacques de Billy [36]. There, he sets out in detail Fermat's methods, which allow him to find as many solutions as necessary in systems of two, three, or more equations. But here his predecessors Diophantus, Bachet and Viet at best found only one solution. After demonstrating Fermat's methods for solving the double equalities Billy also points to the most important conclusion, which follows from this: *This kind of actions serves not only to solve double equalities, but also for any other equations.*

Now it remains only to find out how to use the system of two equations to prove the FLT? Obviously, mathematicians simply did not pay attention to such an explicit clue from Fermat or did not understand its meaning. But for us this is not a problem because we can look into the cache and delve into the "heretical writings"! Based on what we have already been able to recover from Fermat’s works, we can now begin to uncover this greatest mystery of science, indicating also an effective method that allows us to solve the problem of FLT proof.

How it wouldn't be surprising, the essence of this method is quite simple. In the case when there are as many equations as there are unknowns in them, such a system is solved by ordinary substitutions. But if there is only one equation with several unknowns, then it can be very difficult to establish whether it can even have any solutions in integers. In this case, the numbers supposed as

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[47] Similarly, to the example from Pythagoras $3^2+4^2=5^2$ Euler found a very simple and beautiful example of adding powers: $3^3+4^3+5^3=6^3$. For other examples, see comment 22 in Pt. 2.

[48] For example, the task of the infinity of the set of pairs of twin primes or the Goldbach task of representing any even natural number as the sum of two primes. And also, the solution to the coolest problem of arithmetic about an effective way to calculate prime numbers is still very far from perfect despite the tons of paper spent on research on this problem.

[49] In particular, Edwards in his very voluminous book [6, 38], was not aware of the fact that Gauss solved the Fermat's task of decomposing a prime number type 4n + 1 into a sum of two squares. But it was this task that became a kind of bridge to the subsequent discovering the FLT. Fermat himself first reported it in a letter to Blaise Pascal on 09/25/1654 and this is one of the evidences that of all his scientific works, the FLT is really his last and greatest discovery.

solutions can be expressed in the form of another equation called the "Key Formula" and then the result can be obtained by solving a system of two equations. Similar techniques when some numbers are expressed through others, have always been used by mathematicians, but the essence of the key formula is in another, it forms exactly that number, which reflects the essence of the problem and this greatly simplifies the way to solving the original equation. In such approaches and methods, based on an understanding the essence of numbers, in fact also lies the main superiority of Fermat over other scientists.[50]

To make it possible to follow the path that Fermat once laid, you need to find the starting element from the chain of events leading to the appearance of the FLT, otherwise there will be very little chance of success because everything else is already studied far and wide. And if we ask the question exactly this way, we will suddenly find that this very initial element was still in sight from 1670, but since then no one has paid any attention to it at all. However, in fact, we are talking about the very problem under number 8 from the book II of Arithmetic by Diophantus, to which also Fermat's remark was written, became later a famous scientific problem. Everyone thought that this simple-looking problem has no difficulties for science and only Fermat had another opinion and worked for many years to solve it. As a result, he not only obtained it, but in addition to this he secured to his name unfading world fame.

4.1.2. Diophantus' Task

The book entitled "Arithmetic" by Diophantus is very old however, probably it appeared not in III as it was thought until recently, but in the XIV or XV century. In those times when yet there were no print editions, it was a very impressive in volume manuscript consisting of 13 books, from which only six ones reached us. In today's printed form this is a small enough book with a volume of just over 300 pages [2, 27].

In France an original Greek version of this book was published in 1621 with a Latin translation and comments from the publisher, which was Bachet de Méziriac. This publication became the basis for Fermat's work on arithmetic. The contents of the book are 189 tasks and solutions are given for all them. Among them are both fairly simple and very difficult tasks. However, since they have been solved, a false impression is created that these tasks are not educational, but rather entertaining ones i.e., they are needed not to shape science, but to test for quick wit. In those times, it could not have been otherwise because even just literate people who could read and write were very rare.

However, from the point of view the scientific significance of the presented tasks and their solutions, the creation of such a book is not something that to the medieval Diophantus, but to all scientists in the entire visible history would be absolutely impossible. Moreover, even at least properly understanding the contents of the Euclid's "Elements" and the Diophantus' "Arithmetic" became an impossible task for our entire science. Then naturally, the question arises how did the authors of these books manage to do such creations? Of course, this question also arose in science, but instead of answering it only retains its proud silence. Well, then nothing prevents us from expressing our version here.

Apparently, there were somehow preserved and then restored written sources of knowledge from a highly developed civilization perished in earlier times. Only especially gifted people with extrasensory abilities allowing them to understand written sources regardless of the carrier and

[50] The main and fundamental difference between Fermat's methods and the ones of other scientists is that his methods are universal enough for a very wide range of problems and are not directly related to a specific task. As a rule, attempts to solve a problem begin with trial calculations and enumeration of all possible options and those who think faster get correspondingly more opportunities to solve it. Fermat has another approach. He makes trying only for the purpose of bringing them to some universal method suitable for the given task. And as soon as it him succeeds, the task is practically solved and the result is guaranteed even if there is still a very large amount of routine calculations ahead. See for example, comment 30 in Pt. 2.

language, in which they were presented, could read and restore them. Euclid who was most likely a king, involved a whole team of such people, while Diophantus coped itself one and so the authorship of both appeared although in fact it was not the scientists who worked on the books, but only scribes and translators. But now we come back to the very task 8 from the second book of “Arithmetic” by Diophantus: *Decompose a given square into the sum of two squares.*

In the example of Diophantus, the number 16 is divided into the sum of two squares and his method gives one of the solutions $4^2=20^2/5^2=16^2/5^2+12^2/5^2$ as well as countless other similar solutions[51]. However, this is not a solution to the task, but just a proof that any integer square can be made up of two squares any number of times either in integer or in fractional rational numbers. It follows that the practical value of the Diophantus method is paltry since from the point of view of arithmetic, the fractional squares are nonsense like, say, triangular rectangles or something like that. Obviously, this task should be solved only in integers, but Diophantus does not have such a solution and of course, Fermat seeks to solve this problem himself especially since at first, he sees it as not at all complicated.

So, let in the equation $a^2+b^2=c^2$ given the number *c* and you need to find the numbers *a* and *b*. The easiest way to find a solution is by decomposing the number c into prime factors: $c=pp_1p_2\ldots p_k$; then

$$c^2=p^2p_1^2p_2^2\ldots p_k^2=p^2(p_1p_2\ldots p_k)^2=p_i^2N^2$$

Now it becomes obvious that the number c^2 can be decomposed into $a^2 + b^2$ only if at least one of the numbers p_i^2 also decomposes into the sum of two squares.[52] But this is a vicious circle because again you need to decompose square into a sum of two squares. However, the situation is already completely different because now you need to decompose a square of *prime* number and this circumstance becomes the basis for solving the task. If a solution is possible, then there must exist such prime numbers that decomposes into the sum of two squares and only in this case in accordance with the identity of the Pythagoreans, you can obtain:

$$p_i^2=(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$$

i.e. the square of such a prime will also be the sum of two squares. From here appears the truly grandiose scientific discovery of Fermat:[53]

[51] The original solution to the Diophantus' task is as follows. “Let it be necessary to decompose the number 16 into two squares. Suppose that the 1st is x^2, then the 2nd will be $16-x^2$. I make a square of a certain number x minus as many units as there are in site of 16; let it be 2x-4. Then this square itself is $4x^2-16x+16$. It should be $16-x^2$. Add the missing to both sides and subtract the similar ones from the similar ones. Then $5x^2$ is equal to 16x and x will be equal to 16 fifths. One square is 256/25 and the other is 144/25; both folded give 400/25 or 16 and each will be a square” [2, 27].

[52] If $c^2= p^2N^2$ and p^2 (as well as any other p_i^2 of prime factors c) does not decomposed into a sum of two squares i.e. $p^2=q^2+r$ where r is not a square then $c^2=p^2(q^2+r)=(pq)^2+p^2r$ and here in all variants of numbers q and r it turns out that p^2r also is not a square then the number c^2 also cannot be the sum of two squares.

[53] This discovery was first stated in Fermat’s letter to Mersenne dated December 25, 1640. Here, in item 2-30 it is reported: “*This number (a prime of type 4n+1) being the hypotenuse of one right triangle, its square will be the hypotenuse of two, cube – of three, biquadrate – of four etc. to infinity*". This is an inattention that is amazing and completely unusual for Fermat, because the correct statement is given in the neighbor item 2-20. The same is repeated in Fermat’s remark on Bachet’s commentary to task 22 book III of Arithmetic by Diophantus. But here immediately after this obviously erroneous statement the correct one follows: “*This a prime number and its square can be divided into two squares in only one way; its cube and biquadrate only two; its quadrate-cube and cube-cube only three, etc. to infinity*" (see Pt. 3.6). In this letter Fermat apparently felt that something was wrong here, therefore he added the following phrase: “*I am writing to you in such a hurry that I do not pay attention to the fact that there are errors and omit a lot of things, about which I tell you in detail another time*”. This of course, is not that mistake, which could have serious consequences, but the fact is that this blunder has been published in the print media and Internet for the fourth century in a row! It turns out that the countless publications of Fermat's works no one had ever carefully read, otherwise one else his task would have appeared, which obviously would have no solution.

All primes of type 4n+1 can be uniquely decomposed into the sum of two squares, i.e. the equation $p=4n+1=x^2+y^2$ *has a unique solution in integers. But all other primes of type 4n−1 cannot be decomposed in the same way.*

In the Fermat's letter-testament it was shows how this can be proven by the descent method. However, Fermat's proof was not preserved and Euler who solved this problem had to use for this all his intellectual power for whole seven years.[54] Now already the solution to the Diophantine task seems obvious. If among the prime factors of number c there is not one related to the type 4n+1, then the number c^2 cannot be decomposed into the sum of two squares. And if there is at least one such number p_i, then through the Pythagoreans' identity it can be obtain:

$c^2= N^2p_i^2= (Nx)^2+(Ny)^2$

where $x= u^2-v^2$; $y=2uv$; $a=N(u^2-v^2)$; $b=N2uv$

The solution is obtained, however it clearly does not satisfy Fermat because in order to calculate the number N you need to decompose the number c into prime factors, but this task at all times was considered as one of the most difficult of all problems in arithmetic.[55] Then you need to calculate the numbers x, y i.e. solve the problem of decomposing a prime of type 4n+1 into the sum of two squares. To solve this problem, Fermat worked almost until the end of his life.

It is quite natural that when there is a desire to simplify the solution of the Diophantine task, a new idea also arises of obtaining a general solution of the Pythagoras' equation $a^2 + b^2 = c^2$ in a way different from using the identity of Pythagoreans. As it often happens, a new idea suddenly arises after experienced strong shocks. Apparently, this happened during the plague epidemic of 1652 when Fermat managed to survive only by some miracle, but it was after that when he quite clearly imagined how to solve the Pythagoras' equation in a new way.

However, the method of the key formula for Fermat was not new, but when he deduced this formula and immediately received a new solution to the Pythagoras equation, he was so struck by this that he could not for a long time come to oneself. Indeed, before that to obtain one solution, two integers must be given in the Pythagoreans' identity, but with the new method, it may be obtained minimum three solutions with by only one given integer.

But the most surprising here is that the application of this new method does not depend on the power index and it can be used to solve equations with higher powers i.e. along with the equation $a^2+b^2=c^2$ can be solved in the same way also $a^n+b^n=c^n$ with any powers n>2. To get the final result, it remained to overcome only some of the technical difficulties that Fermat successfully dealt with. And here such a way it appeared and became famous his remark to the task 8 of Book II Diophantus' "Arithmetic":

Cubum autem in duos cubos, aut quadrato-quadratum in duos quadrato-quadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duas eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

See Pic. 3 and the translation at the end of Pt. 1.

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[54] Euler's proof is not constructive i.e. it does not provide a method for calculating the two squares that make up a prime of type 4n +1 (see Appendix III). So far, this problem has only a Gauss' solution, but it was obtained in the framework of a very complicated system "Arithmetic of Deductions". The solution Fermat reported is still unknown. However, see comment 172 in Appendix IV (Year 1680).

[55] Methods of calculating prime numbers have been the subject of searches since ancient times. The most famous method was called the "Eratosthenes' Sieve". Many other methods have also been developed, but they are not widely used. A fragment of Fermat's letter with a description of the method he created, has been preserved the letter LVII 1643 [36]. In item 7 of the letter-testament he notes: "*I confess that my invention to establish whether a given number is prime or not, is imperfect. But I have many ways and methods in order to reduce the number of divides and significantly reduce them facilitating usual work*." See also Pt. 5.1 with comments 73-74.

4.2. Fermat's Proof

The reconstructed FLT proof presented here contains new discoveries unknown to today's science. However, from this it does not follows that proof becomes difficult to understand. On the contrary, it is precisely these discoveries that make it possible to solve this problem most simply and easily. The phenomenon of the unprovable FLT itself would not have appeared at all if the French Academy of Sciences had been founded during the lifetime of P. Fermat. Then he would become an academician and published his scientific researches and among his theorems in all arithmetic textbooks there would be also such a most ordinary theorem:

For any given natural number n>2*, there is not a single triplet of natural numbers* a, b and c, *satisfying the equation*

$a^n+b^n=c^n$ (1)

To prove this statement, suppose that a, b, c satisfying to (1) exist and then based on this, we can get the all without exception solutions to this equation in general form. To this aim we use the key formula method, in which one more equation is added to the initial equation so that it becomes possible to obtain solution (1) in a system of two equations. In our case the key formula is:

$a+b=c+2m$ (2)

where m is a natural number.

To obtain formula (2) we note that $a\neq b$ since otherwise $2a^n=c^n$ what is obviously impossible. Consequently, $a<b<c$ and we can state that $(a^{n-1}+b^{n-1})>c^{n-1}$ whence $(a+b)>c$. Since in (1) cases with three odd a, b, c, as well as one odd and two even are impossible, the numbers a, b, c can be either all even or two odd and one even. Then from $(a + b) > c$ follows formula (2) where the number 2m is even[56].

At first, we verify the effectiveness of the method for the case n = 2 or the Pythagoras' equation $a^2+b^2=c^2$. Here the key formula (2) applies and you can get a solution to the system of equations (1), (2) if you substitute one into another. To simplify it, we will square both sides of (2) to make the numbers in (1) and (2) proportionate. Then (2) takes the form:

$\{a^2+b^2-c^2\}+2(c-b)(c-a)=4m^2$ (3)

Substituting the Pythagoras' equation in (3), we obtain:

$A_iB_i=2m^2$ (4)

where taking into account the formula (2):

$A_i=c-b=a-2m$; $B_i=c-a=b-2m$ (5)

Now we decompose the number $2m^2$ into prime factors to get all the A_iB_i options. For primes m there are always only three options: $1\times2m^2=2\times m^2=m\times2m$. In this case $A_1=1$; $B_1=2m^2$; $A_2=2$; $B_2=m^2$; $A_3=m$; $B_3=2m$. Since from (5) it follows $a=A_i+2m$; $b=B_i+2m$; and from (2) $c=a+b-2m$; then we end up with three solutions:

1. $a_1=2m+1$; $b_1=2m(m+1)$; $c_1=2m(m+1)+1$
2. $a_2=2(m+1)$; $b_2=m(m+2)$; $c_2= m(m+2)+2$ (6)
3. $a_3=3m$ $b_3=4m$; $c_3=5m$

Equations (6) are the solutions of the Pythagoras' equation for any natural number m. If the number m is composite, then the number of solutions increases accordingly. In particular, if m

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[56] Fermat discovered formula (2) after transforming the Pythagoras' equation into an algebraic quadratic equation – see Appendix IV story *Year 1652*. However, an algebraic solution does not give an understanding the essence of the resulting formula. This method was first published in 2008 [30].

consists of two prime factors, then the number of solutions increases to nine[57]. Thus, we have a new way of calculating all without exception triples of Pythagoras' numbers, while setting only one number m instead of two numbers that must be specified in the Pythagoreans identity. However, the usefulness of this method is not limited only to this since the same key formula (2) is also valid for obtaining a general solution of equations with higher powers.

Using the method to obtain solutions of (1) for the case n=2, it is also possible to obtain solutions for n>2 by performing the substitution (1) in (2) and exponentiating n both sides of (2). To do this, first we derive the following formula[58]:

$(x+y)^n=z^n=zz^{n-1}=(x+y)z^{n-1}=xzz^{n-2}+yz^{n-1}=$

$x(x+y)z^{n-2}+yz^{n-1}=x^2zz^{n-3}+y(z^{n-1}+xz^{n-2})+\ldots$

$(x\pm y)^n=z^n=x^n\pm y(x^{n-1}+x^{n-2}z+x^{n-3}z^2+\ldots+xz^{n-2}+z^{n-1})$ (7)

We will name the expression in brackets consisting of n summand a *symmetric polynomial* and we will present it in the form $(x ++ z)_n$ as an abridged spelling. Now using formula (7), we will exponentiating n both sides of formula (2) as follows.

$[a-(c-b)]^n=a^n+\{b^n-c^n+(c^n-b^n)\}-(c-b)[a^{n-1}+a^{n-2}2m+\ldots$

$+ a(2m)^{n-1}+(2m)^{n-1}]=(2m)^n$

Now through identity

$(c^n-b^n)=(c-b)(c^{n-1}+c^{n-2}b+\ldots+cb^{n-2}+b^{n-1})$ we obtain:

$\{a^n+b^n-c^n\}+(c-b)[(c++b)_n-(a++2m)_n]=(2m)^n$ (8)

Equation (8) is a formula (2) raised to the power n what can be seen after substituting c−b=a −2m in (8) and obtaining the identity[59]:

$\{a^n+b^n-c^n\}+(c^n-b^n)-[a^n-(2m)^n]=(2m)^n$ (9)

In this identity natural numbers a, b, c, n, m of course, may be any. The only question is whether there are such among them that $\{a^n+b^n-c^n\}$ will be zero? However, the analogy with the solution of the Pythagoras' equation ends on this since the substitution of (1) in (8) is not substantiated in any way. Indeed, by substituting (1) in (3), it is well known that the Pythagoras' equation has as much as you like solutions in natural numbers, but for cases n>2 there is no single such fact. Therefore, the substitution of the non-existent equation (1) in (8) is not excluded, what should lead to contradictions. Nevertheless, such a substitution is easily feasible and as a result we obtain an equation very similar to (4), which gives solutions to the Pythagoras equation. Taking into account this circumstance, we yet substitute (1) in (8) as a test, but at the same time modify (8) so, that factor $(c-a)$ take out of square brackets.[60]

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[57] For example, if $m = p_1p_2$ then in addition to the first three solutions there will be others: $A_4=p_1$; $B_4=2p_1p_2^2$; $A_5=p_2$; $B_5=2p_1^2p_2$; $A_6=2p_1$; $B_6=p_1p_2^2$; $A_7=2p_2$; $B_7=p_2p_1^2$; $A_8=p_1^2$; $B_8=2p_2^2$; $A_9=p_2^2$; $B_9=2p_1^2$

[58] Formula (7) is called *Fermat Binomial*. It is curious that the same name appeared in 1984 in the novel "Sharper than the epee" by the Soviet science fiction writer Alexander Kazantsev. This formula is not an identity because in contrast to the identity of *Newton Binomial* in addition to summands, there is also a sum of them, but with the help of *Fermat Binominal* it is easy to derive many useful identities in particular, factorization of the sum and difference of two identical powers [30], see also Pt. 4.4.

[59] In this case, identity (9) indicates that the same key formula is substituted into the transformed key formula (2) or that the equation (8) we obtained, is a key formula (2) in power n. But you can go the reverse way just give the identity (9) and then divide into factor the differences of powers and such a way you can obtain (8) without using the *Fermat Binominal* (7). But this way can be a trick to hide the understanding of the essence because when some identity falls from the sky, it may seem that there is nothing to object. However, if you memorize only this path, there is a risk of exposure in a misunderstanding of the essence because the question how to obtain this identity, may go unanswered.

[60] Taking into account that c−a=b−2m the expression in square brackets of equation (8) can be transformed as follows: $(c++b)_n - (a++2m)_n = c^{n-1}- a^{n-1}+ c^{n-2}b- a^{n-2}2m+ c^{n-3}b^2- a^{n-3}(2m)^2+ \ldots +b^{n-1} - (2m)^{n-1}$; $c^{n-1} - a^{n-1} = (c - a)(c++a)_{n-1}$; $c^{n-2}b - a^{n-2}2m = 2m(c^{n-2} - a^{n-2}) + c^{n-2}(b - 2m) = (c - a)[2m(c++a)_{n-2} + c^{n-2}]$; $c^{n-3}b^2 -a^{n-3}(2m)^2 = (2m)^2(c^{n-3} - a^{n-3}) + c^{n-3}(b^2 - 4m^2) = (c - a)[4m^2(c++a)_{n-3} + c^{n-3}(b +2m)]$; $b^{n-1} - (2m)^{n-1} = (b - 2m)(b++2m)_{n-1} = (c - a)(b++2m)_{n-1}$ All differences of numbers except the first and last, can

Then we obtain:

$A_iB_iE_i=(2m)^n$ (10)

where $A_i = c-b=a-2m$; $B_i=c-a=b-2m$; E_i – polynomial of power $n-2$.

Equation (10) is a ghost that can be seen clearly only on the assumption that the number $\{a^n+b^n-c^n\}$ is reduced when (1) is substituted into (8). But if it is touched at least once, it immediately crumbles to dust. For example, if $A_i\times B_i\times E_i=2m^2\times 2^{n-1}m^{n-2}$ then as one of the options could be such a system:

$A_iB_i=2m^2$

$E_i=2^{n-1}m^{n-2}$

In this case, as we have already established above, it follows from $A_iB_i=2m^2$ that for any natural number m the solutions of equation (1) must be the Pythagoras' numbers. However, for n>2 these numbers are clearly not suitable and there is no way to check any other case because in a given case (as with any other variant with the absence of solutions) another substitution will be definitely unlawful and the ghost equation (10), from which only solutions can be obtained, disappears.[61] Since the precedent with an unsuccessful attempt to obtain solutions has already been created, there can be no doubt that also all other attempts to obtain solutions from (10) will be unsuccessful because at least in one case the condition $\{a^n+b^n-c^n\}=0$ is not fulfilled i.e. the equation (10) has been obtained by substituting a non-existing (1) in the key formula (2), and the Fermat's Last Theorem is proven.[62]

So, now we have a restored author's proof of the Fermat's most famous theorem. Here are interesting ideas, but at the same time there is nothing that could not be accessible to science for more than three hundred years. Also, from the point of view a difficulty in understanding its essence, it matches at least to the 8th grade of secondary school. Undoubtedly, the FLT is a very important component of number theory. However, there is no apparent reason that this task has become an unsolvable problem for centuries, even though millions of professional scientists and amateurs have taken part in the search for its solution. It remains now only to lament, that's how he is, this unholy!

After everything was completed so well with the restoration of the FLT proof, many will be disappointed because now the fairy tale is over, the theme is closed and nothing interesting is left here. But this was the case before, when in arithmetic there were only rebuses, but we know that this is not so, therefore for us the fairy tale not only has not ended, but even did not begin! The fact is that we have so far revealed the secret of only two of the Fermat's six recordings, which we have restored at the beginning of our study. To make this possible, we made an action-packed historical

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be set in general form: $c^xb^y - a^x(2m)^y=(2m)^y(c^x - a^x) + c^x[b^y - (2m)^y] = (c - a)(c++a)_x(2m)^y + (b - 2m)(b++2m)_yc^x = (c - a)[(c++a)_x(2m)^y + (b++2m)_yc^x]$ And from here it is already become clear how the number $(c - a)$ is take out of brackets. Similarly, you can take out of brackets the factor $a + b = c + 2m$. But this is possible only for odd powers n. In this case, equation (10) will have the form $A_iB_iC_iD_i = (2m)^n$, where $A_i = c - b = a - 2m$; $B_i = c - a = b - 2m$; $C_i = a + b = c + 2m$; D_i – polynomial of power $n - 3$ [30].

[61] Equation (10) can exist only if (1) holds i.e. $\{a^n+b^n-c^n\}=0$ therefore, any option with no solutions leads to the disappearance of this ghost equation. And in particular, there is no "refutation" that it is wrong to seek a solution for any combination of factors, since $A_iB_i=2m^2$ may contradict $E_i=2^{n-1}m^{n-2}$, when equating E_i to an integer does not always give integer solutions because a polynomial of power $n-2$ (remaining after take out the factor $c-a$) in this case may not consist only of integers. However, this argument does not refute the conclusion made, but rather strengthens it with another contradiction because E_i consists of the same numbers (a, b, c, m) as A_i, B_i where there can be only integers.

[62] In this proof, it was quite logical to indicate such a combination of factors in equation (10), from which the Pythagoras' numbers follow. However, there are many other possibilities to get the same conclusion from this equation. For example, in [30] a whole ten different options are given and if desired, you can find even more. It is easy to show that Fermat's equation (1) is also impossible for fractional rational numbers since in this case, they can be led to a common denominator, which can then be reduced. Then we get the case of solving the Fermat equation in integers, but it has already been proven that this is impossible. In this proof of the FLT new discoveries are used, which are not known to current science: there are the key formula (2), a new way to solve the Pythagoras' equation (4), (5), (6), and the Fermat Binomial formula (7) … yes of course, else also magic numbers from Pt. 4.3!!!

travel, in which the LTF was an extra-class guide. This travel encouraged us to take advantage of our opportunities and look into these forbidden Fermat's "heretical writings" to finally make a true science in image of the most fundamental discipline of arithmetic available to our intelligent civilization and allowing it to develop and flourish on this heavy-duty foundation like never before.

We can honestly confess that so far not everything that keep in Fermat's cache is accessible and understandable to us. Moreover, we cannot even determine where this place is. But also, to declare that everything that we tell here, is only ours, would be clearly unfair and dishonest because nobody would have believed us then. On the other hand, if everything was so simple, then it would be completely to no one interesting. The worst thing that could be done, is to reveal the entire contents of Fermat's cache so that everyone will forget about it immediately after reading.

We will act otherwise. If something will be revealed, only to give an opportunity to learn about the even more innermost mysteries of science, which will not only make everyone smarter, but will indicate the best ways to solve vital problems. Using the example of solving the FLT problem, it will be quite easily to make sure this since with a such solution science receives so a reliable point of support that it can do whatever it wants with the integer power numbers. In particular, it may be easily calculated as much as you like of integer power numbers, which in sum or difference will give again an integer power number. The fact that only a computer can shovel such a work, is very ashamed for current science because this task is too simple even for children.

The most quick-witted of them will clearly prefer that adults ask them to explain something more difficult for example, FLT proof, which in their time was completely inaccessible to them. Children of course, will not fail to get naughty and will be important like high-class nobles when answer to stupid questions of adults and indicating to them that it would be nice for someone to learn something else. But it will be only little flowers. But after that, the amazement of adults will become simply indescribable when they find out that the children are addicted to peeping and copying everything that interests them directly from Fermat's cache! Indeed, at their age they still do not realize their capabilities and it seems to them that this is at all not a difficult task.

However, if they had not read interesting books about science, then such an idea would never have occurred to them. But when they find out that someone is doing this, they will find that they can do it just as well if even not better! Do you not believe? Well, everyone who wants to be convinced of this, will have this opportunity now. But one more small detail remains. Fermat in his "heretical writings" although he pointed out that he had to provide proofs of three simple theorems for children, which he specially prepared for them, but so far he did not have time for this nevertheless firmly promised that as soon as he has a time, then he will certainly and sure do it.

But apparently, he did not have enough time and so he did not manage to add the necessary recordings. Or perhaps he changed his mind because didn't want to deprive children of joy on their own to learn to solve just such problems that adults can't afford. If the children can't cope, then who them will reproach for it. But if they manage it, then adults will not go anywhere and will bring to children many, many gifts!

4.3. Theorems About Magic Numbers

The above presented proof of FLT not only corresponds to Fermat's assessment as" truly amazing", but is also constructive since it allows us to calculate both the Pythagorean numbers and other special numbers in a new way what demonstrate the following theorems.

Theorem 1. *For any natural number n, it can be calculated as many triples as you like from different natural numbers a, b, c such that* $n = a^2 + b^2 - c^2$. For example :

$n=7=6^2+14^2-15^2=28^2+128^2-131^2=568^2+5188^2-5219^2=$
$=178328^2+5300145928^2-5300145931^2$ etc.

$n=34=11^2+13^2-16^2=323^2+3059^2-3076^2=$
$=247597^2+2043475805^2-2043475820^2$ etc.

The meaning of this theorem is that if there is an infinite number of Pythagoras triples forming the number zero in the form $a^2+b^2-c^2=0$ then nothing prevents creating any other integer in the same way. It follows from the text of the theorem that numbers with such properties can be “calculated”, therefore it is very useful for educating children in school.

In this case, we will not act rashly and will not give here or anywhere else a proof of this theorem, but not at all because we want to keep it a secret. Moreover, we will recommend that for school books or other books (if of course, it will appear there) do not disclose the proof because otherwise its educational value will be lost and children who could show their abilities here will lose such an opportunity. On the other hand, if the above FLT proof would remain unknown, then Theorem 1 would be very difficult, but since now this is not so, even not very capable students will quickly figure out how to prove it and as soon as they do, they will easily fulfill the given above calculations.

Конец ознакомительного фрагмента.