

# PROBABILISTIC THEORY OF STOCK EXCHANGES





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**Probabilistic Theory  
of Stock Exchanges**

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This monograph presents the foundations of the probabilistic theory of stock exchanges, built on the basis of probabilistic economic theory. Analytical and numerical methods of this theory are used for calculation of temporal dynamics of market prices and trade volumes of various assets on the Moscow Exchange and Intercontinental Exchange Futures Europe during one trading session and a detailed comparison of the theoretical results with the corresponding experimental data. This comparison demonstrates a good agreement between the theory and experiment, which allows us to assert that the main scientific problem of this study is solved in the monograph, namely, it is shown that probabilistic economic theory finds its experimental confirmation and thereby acquires its solid experimental justification. This monograph may seem interesting to everyone who is engaged in research in economics, finance, econophysics or physical economics, as well as to professional investors and stock traders.

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# Anatoly Kondratenko

## Probabilistic Theory of Stock Exchanges

*To my beloved parents Vasily Denisovich and Alexandra Vasilevna  
Kondratenko with appreciation and gratitude*

### PREFACE

This is the third monograph in a series devoted to the development of probabilistic economic theory [Kondratenko, 2005, 2015, 2021]. When the first book was being prepared for publication, the publisher asked the author how soon, in the author's opinion, this new theory would gain widespread acceptance among economists. His answer was: at the very least 20 years, for the simple reason that it is a priori not enough to put forward another alternative economic theory – first, it is necessary to show its benefits as compared to the others, second – to prove that it is correct, and third – to showcase how it can be used in practice to describe experimental data and develop predictions of economic dynamics. This monograph addresses all three of these issues: it shows how this theory is better than others; it proves, by the example of important organized markets such as stock exchanges, that it is correct; and, based on the analytical and numerical results obtained, it offers a method for describing and forecasting economic dynamics.

It is explained below why stock markets were chosen as the main object of study. It is well known that specialized activities of goods exchange or trade between different producers and different countries plays an eminent role in the growth of the welfare of individuals and states as a whole in the early stages of its development, and then in the subsequent rapid formation and development of capitalism in the world. In this process, the most important place was first occupied by ordinary markets, which facilitated and thus accelerated the process of exchange of some goods or values for others, and then by organized markets, the highest form of which are, in particular, stock exchanges: commodity, financial, currency and others, which form and represent in their entirety the stock exchange economy. This exchange economy is now effectively using all the most advanced methods and tools of electronic trading, including artificial intelligence and algorithmic trading using computers; while the extensive use of the Internet makes the exchange economy very fast, virtual and truly global. It is the high speed of information exchange and transactions that distinguishes the new virtual exchange economy from the traditional real economy. But this specific feature of exchange trading can carry additional risks for both the exchange and the real economy, the tasks of which are still rather superficially understood in the economic academic community. In today's global economic world, the role of stock exchanges has become so significant that it would not be an exaggeration to say that the entire global economy is gradually becoming exchange-based. For sure, it would be more accurate to talk about the transformation of the global economy into a financial economy, but this monograph will be focused on studying only the role of exchanges in the economy, so the term “exchange economy” will be used. The main purpose of exchanges in today's economy is to determine the prices of all traded assets, including various money (currencies), to facilitate their trade and provide financing for the global economic activity. But it is also important that the situation in exchanges is the most universal indicator of the situation in the entire global economy. The paradox of this situation in this global financial world is that there is clearly no correlating situation in the world of theoretical finance. An adequate theory of organized financial markets still does not exist, which means that an adequate theory of the global market real economy is also out of question. This situation gives rise to certain risks of the emergence and uncontrolled development of negative trends in financial markets, which can lead to large-scale financial and then economic crises. And that's

what we regularly observe in real life: the generator of almost all economic crises in modern history are financial crises triggered by a stock market crash.

Currently, the situation is aggravating and the risks are increasing due to the fact that the bulk of transactions are now carried out by computers, working strictly according to algorithms, aimed mainly at achieving quick results without even minimal losses, and acting almost simultaneously, which can cause a chain collapse at stock exchanges uncoupled from the actual situation in the economy and the real value of assets. Meanwhile, the regulators do not have any meaningful and reliable tools for controlling and managing particularly explosive situations in financial markets, especially in organized markets or stock exchanges, where the prices of all global commodities and assets are largely determined. Such regulating agencies currently use accumulated historical experience and empirical parametric models to develop their management processes [Intriligator, 2002]. For these reasons, overcoming the apparent stagnation in the development of theoretical finance is a long overdue global task. The main challenge here and now is the almost complete absence of a mathematical apparatus with a potential to be used not only describe the exchange functioning as an asset pricing mechanism, which is done by financial econometrics at a qualitative level, but also accurately calculate the temporal fine price structure and the temporal fine structure of the trade volume during short time intervals, for example, during one trading session. Using the analogy with the scattering theory in physics, this can be formulated the other way around. Econometrics solves the so-called inverse problem, namely, extracting information about the studied system from the experimental data. The present study, however, aims at solving a direct problem – creating a fairly universal method for ab initio calculation of the exchange time microstructures with a characteristic size of a few seconds, which can be directly compared with the corresponding experimental fine structures of time trading dynamics, a method that could serve as a powerful tool for building a general theory of exchanges. We hope that the probabilistic theory of stock exchanges developed in this study can serve as a basis for building a general probabilistic financial theory and a deeper understanding of how the global world of finance works.

Obviously, organized markets are complex multiagent non-equilibrium probabilistic systems, and their description requires adequate mathematical methods. At the moment, the only suitable source of such methods is, of course, physics, which has long accumulated experience in theoretical work with similar multiparticle systems. Besides, quite a lot of experience has already been accumulated with regard to the physical methods application in economics, i.e., the use of formal methods and approaches of theoretical physics to solve economic problems. In particular, in the works of A.V. Kondratenko [2005, 2015] a new theory of market economy – Probabilistic Economic Theory – was developed. Initially, this theory was based on quantum mechanics with the derivation of economic equations of motion, which have not yet been solved for multiagent markets. Subsequently, a simpler version of the theory was developed using the probabilistic method, namely – probability economy or probabilistic economics, which was used as the basic theory to develop the probabilistic theory of the exchange in this paper, although it does not have motion equation, but has a mathematical framework, which proved very adequate and viable for the description of exchange processes and structures. To be on the safe side and avoid misunderstandings, let us clarify that there is literally no physics in this theory, no mechanics, let alone quantum mechanics. It is an economic theory to describe the economic processes that take place in the stock exchange. It simply adapts to economics mathematical tools that were created hundreds of years ago and were previously successfully used to solve similar problems in physics.

Probabilistic economics can be seen as a theory developed in the manner of general ideology as a physical method in economics, as well as in classical economic theory, understanding it as a version of the economic theory development, which can be figuratively described by an evolutionary trajectory: Adam Smith – Karl Menger – Ludwig von Mises. The works of these three great economists gave rise to the author's understanding of the essence and purpose of real economic

science and the desire to develop their ideas and concepts by using the modern scientific probabilistic method of research. The primary task then was to create a mathematical apparatus or body equivalent to the physical method and use it to calculate specific real economic systems similar to how the same process worked in the history of the development and rapid rise of physical science, due to the creation of a powerful mathematical body, starting with the discovery of the equations of motion and differential calculus.

The probabilistic method has long been used in economic research at the empirical level using the basic formulas of probability theory. The use of this method in economics, by analogy with the quantum mechanics of physical systems [Kondratenko, 2005, 2015], has widely expanded the scope of our ideas about the modern economic world, generated a new probabilistic style of scientific economic thinking and created a new probabilistic dynamic picture of the modern economic world, alternative to traditional static ideas of mainstream economics, including neoclassical economics. This monograph essentially solves the problem of practical application of this approach to specific economic systems, namely stock exchanges, since we have enough input data for their quantitative study, i.e., supply and demand quotations, as well as relevant experimental data represented by market prices and trade volumes for theory verification.

Probabilistic economics, as well as many probabilistic theories in various fields of science, primarily in physics, for example, in statistical and quantum physics, is developed in terms of probability distributions. We emphasize that probability distributions are the very thing that forms the basis and language of a probabilistic, scientifically substantiated method for studying complex dynamic systems.

The possibilities of probabilistic economics methods to accurately describe real markets have been demonstrated earlier [Kondratenko, 2005, 2015] using examples of small model commodity economies. In this work the author, on the basis of probabilistic economics, aimed to create the foundations of the stock exchange theory, capable of filling the gaps described above in the modern theory of finance, the results of which will be in good agreement with the experimental stock exchange data. This goal was achieved. Let us specify that it is clear from general considerations that the microscopic theory developed in this study is devoted to the study of various stock exchange structures and processes at the level of exchange agents, and even more precisely – at the level of actions of individual exchange agents. The main purpose of microscopic theory is to describe the process of these structures' formation based on the specific actions of the agents. First of all, this study will cover the mechanisms of exchange prices and trade volumes formation based on the quotations of market agents (in a particular period of time). It can be figuratively said that this theory gives a microscopic view of the stock exchange and stock exchange phenomena.

The book will show that probabilistic economics provides an adequate, fairly accurate micro- and macroscopic description of the stock exchange, namely, detailed structures and mechanisms of its operation, which were studied to derive certain patterns in the work of stock exchanges, in particular patterns in the prices and trade volumes formation.

We emphasize that for the purposes of this study we made no difference between stock, commodity, currency and other exchanges; the theory developed is equally suitable for describing any exchange where assets are traded, so for brevity in this book we will talk about stock exchanges, or simply exchanges.

As a conclusion, we provide a summary of the monograph with a subjective evaluation of the results obtained and the conclusions drawn in the study. The book outlines the basics of the probabilistic theory of stock exchanges, built on the basis of probabilistic economic theory using agent quotations provided by stock exchanges. By its nature, this theory of exchanges is microscopic, so its analytical and numerical methods make it possible to calculate and describe various exchange microstructures and microprocesses. The calculations of this type were performed for the first time in this study and are also published for the first time. The main attention was paid to the calculation

of market prices and trade volumes of various assets (Sberbank shares, Brent oil futures, American dollars) on the Moscow Exchange and Intercontinental Exchange Futures Europe (Brent oil futures) during one trading session, as well as a detailed comparison of the calculation results with the experimental data. This comparison demonstrates a good agreement between theory and experiment, which allows us to state that the monograph has fulfilled the main scientific purpose of the proposed study, namely, to show that the probabilistic economic theory finds experimental confirmation and, thus, acquires solid experimental justification.

This sets it apart from other economic theories of a heuristic or empirical nature. Another important purpose has also been fulfilled, namely to describe in detail the economic mechanism underlying the formation of market prices and trade volumes and serving as a bridge connecting the microscopic and the macroscopic economic world, thus demonstrating the process of macrocosm formation from microcosm, namely how the actions of exchange agents form the action and temporal dynamics of the market as a whole.

A new, universal system of exchange indices of assets, stock exchanges and the global system of exchanges has been developed. By analogy, the strategy of digitalization, forecasting and management of economies on the basis of digital platforms was developed for accumulating plans of economic agents and processing them using the formulas of probabilistic economic theory, which, if implemented, will in turn improve the quality of public administration of the economy of individual countries and the world as a whole.

The monograph demonstrates the importance and significance of stock exchanges as experimental economic laboratories, aimed primarily at testing models, evaluating parameters of models and, ultimately, verifying the existing and new economic theories. While the construction of probabilistic economic theory as an «empirical» laboratory was to a certain extent backed up by the business, the practical 25-year experience of management of which was formalized by the mathematical body of theoretical physics, the development of probabilistic theory of stock exchanges in the study was based on MOEX and ICE as experimental economic laboratories. Step by step, the monograph reveals the enormous prospects for the further use of stock exchanges as powerful up-to-date experimental economic laboratories, which allow us to argue that just as theoretical physics emerged from the science of the solar system some 300–500 years ago, in the near future a modern economic science, consisting of closely interacting theoretical economics and experimental economics, will emerge from the development of stock exchange theory based on exchange experiments that will match all generally accepted standards for physical sciences, while remaining a human and social science.

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In conclusion, the author would like to take the opportunity, unfortunately, very late, to pay back some of his old debts.

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## FREQUENTLY USED SYMBOLS

$\Gamma$  – agent width

MOEX – Moscow Exchange

BRENT – Brent oil futures

$C$  – normalization factors

$D$  – demand

$D(t, p, q)$  – probabilistic market demand function

$D^0(t)$  – total market demand function

ICE – Intercontinental Exchange Futures Europe

$F(t, p, q)$  – probabilistic market deal function

$M(t)$  – number of supply quotations

$N(t)$  – number of demand quotations

$MTV(t)$  – probabilistic market trade volume

$P$  – price

$p$  – independent price variable

$p^M$  – probabilistic market price

$q$  – independent quantity variable

$q^M$  – probabilistic market quantity

$Q$  – quantity

$PQ$  – price and quantity

$S$  – supply

SBER – Sberbank shares

$S D$  – supply and demand

$S(t, p, q)$  – probabilistic market supply function

$S^0(t)$  – total market supply function

$t$  – independent time variable

$TV(t)$  – trade volume

$T$  – time

USD/RUB – USD/RUB futures at MOEX: U.S. dollars are traded for Russian rubles (#).

## INTRODUCTION

"The impartial observer can have no doubt about the reason our generation pays general and enthusiastic tribute to progress in the field of the natural Sciences, while economic Science receives little attention and its value is seriously questioned by the very men in society to whom it should provide a guide for practical action. Never was there an age that placed economic interests higher than does our own. Never was the need of a scientific foundation for economic affairs felt more generally or more acutely. And never was the ability of practical men to utilize the achievements of Science, in all fields of human activity, greater than in our day. If practical men, therefore, rely wholly on their own experience, and disregard our Science in its present state of development, it cannot be due to a lack of serious interest or ability on their part. Nor can their disregard be the result of a haughty rejection of the deeper insight a true Science would give into the circumstances and relationships determining the outcome of their activity. The cause of such remarkable indifference must not be sought elsewhere than in the present state of our Science itself, in the sterility of all past endeavors to find its empirical foundations. The reason for this conspicuous indifference is none other than the present state of science itself, the fruitlessness of hitherto attempts to comprehend its empirical foundations".

*Carl Menger* [2007].

"Next, the empirical background of economic science is definitely inadequate. Our knowledge of the relevant facts of economics is incomparably smaller than that commanded in physics at the time when the mathematization of that subject was achieved. Indeed, the decisive break which came in physics in the seventeenth century, specifically in the field of mechanics, was possible only because of previous developments in astronomy. It was backed by several millennia of systematic, scientific, astronomical observation, culminating in an observer of unparalleled caliber, Tycho de Brahe. Nothing of this sort has occurred in economic science. It would have been absurd in physics to expect Kepler and Newton without Tycho, – and there is no reason to hope for an easier development in economics".

*John Von Neumann and Oskar Morgenstern* [1970]

The founding fathers of the Austrian school of economics established economic theory on a solid empirical footing in their day, which predetermined its successful development for many years to come. But the current levels of rigor of the underlying concepts and assumptions of these theories, as well as the quantitative description of real economic processes and phenomena, not to mention the quality of economic forecasts, are clearly insufficient for developing an evidence-based management of countries' economies and achieving sustainable development of the global economy. There is a huge gap between the modern requirements that the society in its wide understanding presents to the economic science and the ability of this science to meet such requirements. This, as 150 years ago, generates a negative, at best ironic, public attitude to economic science, existing in the form of a number of often mutually exclusive theories, such as neoclassical economics and the Austrian school of economics (below often just Austrian economics), whose adherents give contradictory estimates, forecasts and recommendations. It has now come to the realization that empirical foundations alone are clearly insufficient for the verification of adequate models and approaches to economics. It is time to make a strict selection among all existing theories and currents of economic thought by means of their experimental verification in order to further develop economic theory, capable of providing a quantitative description of economic processes and phenomena at a high scientific level, comparable with the level of research in the natural sciences. As a result of this selection, economic theory will get a solid experimental foundation and become a unified economic science, like physics and

other natural sciences, rather than a stream of ten parallel currents competing for financial resources, represented by neoclassical economics, Keynesianism, Marxism, etc.

To be clear, let us emphasize once again that all currently wide known economic theories, including neoclassical economics as the mainstream, are essentially heuristic or, at best, empirical theories built upon observation of the economic activities of market agents and the state, as well as on collecting various economic facts and their subsequent verbal generalization into a set of formulated principles for the economic activities of people and enterprises as well as the economic policies of the state. Not surprisingly, therefore, the theories of finance derived from them are extremely limited in their ability to quantitatively describe the temporal fine structure of the dynamics of ordinary and even more so of organized markets both because of our limited knowledge of the general economic laws governing the functioning of markets and because of an almost complete lack of a mathematical body which could be used to calculate at the microscopic level the temporal fine structure of markets in small time intervals, such as one trade session, and then to discover new patterns of how the markets work, using detailed comparison of the obtained results with the experimental data.

In this book we are committed to consistently overcome the above problems within the framework of probabilistic economics according to the following program of actions: developing a mathematical apparatus for calculating *ab initio* (from the first principles) the temporal fine structure of organized financial markets or exchanges, determining patterns in the functioning of financial markets obtained by comparing theoretical results with experimental data provided by exchanges, and eventually determining the patterns governing these markets, in the strict mathematical language. The patterns thus obtained can be used to derive motion equations that describe the temporal dynamics of market economic systems, in other words, equations that describe the evolution of economies. Thus, the purpose of this study is to create a theory of organized markets that has a sound experimental foundation. Meanwhile, if this venture proves successful, it could be argued that the previously constructed probabilistic economic theory also received a solid experimental foundation.

Using the analogy with the theory of scattering, we can say that probabilistic economics is aimed at solving the direct problem of economics, namely, based on some general principles, to calculate the results of economic activity or economic experiments and to compare them directly with the corresponding experimental data, which will allow to obtain a reliable interpretation of experimental data. At the same time, econometric studies solve the opposite problem – to extract information about the properties of the studied economic system from the experimental data using the help of mathematical methods. In order to avoid misunderstandings, we emphasize that everything that is stated in this monograph, and everything that is asserted in it, unless specifically stated, concerns only the direct problem of economics.

All of the above can be phrased somewhat differently. At the present time, there are two main problems of economic science in the theory of organized markets.

**Problem 1** is the almost complete absence of a mathematical body that would allow us to conduct full-fledged theoretical calculations of the details of the exchange markets temporal dynamics at the microscopic level on a small-time scale, for example within 1 hour or one trading session.

**Problem 2** is the lack of an experimental basis for economic science in the sense understood in natural sciences: data from systematic theoretical calculations should quantitatively coincide with corresponding experimental data with a satisfactory degree of accuracy.

When solving the above problems, we intend to rely on the physical method of economic research, the main feature of which is the aspiration to find and formulate economic laws in the form of equations, to use to the full extend the mathematical body to perform sufficiently accurate quantitative calculations and constantly rely on experiment in verification of hypotheses, theories and concrete numerical results. This method makes it possible to overcome the shortcomings of the simple empirical method, which currently prevails in economic research, based on a logical analysis of experimental data, as, for example, in the Austrian economic school or in econometric analysis



of price dynamics, and to achieve the same level of scientific rigor as in natural sciences, above all physics. Without physical methods of research, i.e., without reliance on experiment, the further development of economic theory is impossible; otherwise, it will long remain in its infancy, in other words, it will remain a kind of protoscience in comparison, say, with natural sciences, above all physics.

First, we will very briefly express a subjective opinion about the state of modern economic science, so that the reader could understand the logic of the research undertaken in this work and its main objectives, and, ultimately, the value of the results obtained. We will formulate our opinion in the form of two statements.

**Statement one.** In our view, all old and new, widely known economic theories, including neoclassical economics, Marxist and Keynesian theories, the Austrian economic school and other currents of economic thought are, in fact, either heuristic or, at best, empirical theories with neither clear unambiguous experimental results, nor rigorous mathematical theories that allow ab initio calculations on the dynamics of specific real market systems whose results coincide with the corresponding experimental results of these markets work with a reliable level of accuracy. Moreover, proponents of even the most logically advanced empirical economic theory, namely the Austrian economic school, argue [Von Mises, 2005; De Soto, 2009] that neither experimentation nor even the use of the mathematical apparatus to describe economic phenomena and market processes is possible in principle. On this basis they categorically denounce all attempts to use the achievements of physics and mathematics for development of the quantitative economic theory. In our opinion, the current situation in economics is not absolute; it only repeats the similar situation that existed in physics 300–600 years ago before the works of Nicholas Copernicus, Isaac Newton, Galileo Galilei and other physicists and mathematicians of the new era in physics. What is the main reason for economics to lag behind physics for so long in this respect? John von Neumann and Oskar Morgenstern provide an excellent answer to this question in the quote from their book given in the epigraph. The reason was hidden in an objective factor, namely in the very absence of the possibility to rely on experiment in economics, at least in the form of systematic long-term observations of the cyclic motion of the planets of the solar system, as was done in physics. At present such an opportunity is provided to us by electronic exchanges with their digital platforms and big data that can be used, in general, for the verification and development of economic theories.

Returning to the beginning of the discussion, let us note that the probabilistic economics we developed was also empirical or heuristic in content, based on our twenty-five years of entrepreneurial experience in the investment business. So, in this respect it is no better than any other economic theory; thus, it is virtually unknown in the scientific economic community. But there is an important nuance. Unlike all other theories, probabilistic economics has a developed mathematical body suitable for calculation of any market economic systems. The results of these calculations can be compared with known experimental data, for example for exchanges. In this regard, in this study we have set a goal to find experimental evidence that the very foundations of probabilistic economics are valid, in other words, to verify the initial premises and assumptions of the theory by means of experiments. Moreover, this should be done the way it is done in physics, namely by continuously comparing the results of ab initio calculations of the dynamics of exchange systems and the results of experiment, as well as by subsequently confirming or rejecting the assumptions made. Only this approach, or method of investigation, which we call the physical method in economic science and which is universally recognized in the natural sciences, will make it possible to develop an adequate economic theory capable of giving a sufficiently accurate quantitative description of how real markets of any complexity work, as well as of making sufficiently reliable forecasts of markets and economies development, at least in the short term. In other words, we aim thereby to establish sufficiently accurate experimental justifications for economic science.

**Statement two.** Let's recall the great importance of the long-term observations of the solar system planets behavior [Smith, 2016] in the development of modern physics and what John von Neumann said (see above). Richard Feynman was of the same opinion: "Astronomy is older than physics. In fact, physics emerged from it when astronomy noticed the striking simplicity of stars and planets motion; the explanation of this simplicity was the beginning of physics." Figuratively speaking, the solar system played the role of the first experimental physical laboratory in the history of science. Of course, it was impossible to perform experiments on it in the modern sense of the word, but it was possible to observe the motion of the planets without interference for a long time, and based on these observations the scientists could try to find the rules governing this motion, and even calculate the trajectories of the planets, which was actually done [Feynman et al., 1978; Smith, 2016]. Fortunately, this movement was frequent enough, almost exactly the same, that it allowed us to observe the same phenomena for quite a long time. And the strict periodicity in the planets motion clearly indicated the existence of strict rules governing this motion. It was just a matter of discovering them.

What about economics? Fortunately, we have at our disposal a wonderful experimental economic laboratory that has the potential to brilliantly play the same role in economics that the solar system has played in physics. These are, of course, the exchanges that determine the market prices of goods, services and, especially important in today's economic world, financial assets of various kinds. Physically speaking, exchanges measure prices at each moment of trading, which are unconditionally accepted by the economic community as market prices, i.e., as valid and fair. By measuring market prices and making them universally available, exchanges play an enormous role in modern economic life, providing everyone with a basis for making crucial economic decisions. Despite the important role of exchanges in the real economy, the importance of exchanges in economic theory is far from significant for the reason that a sufficiently developed theory of the exchange capable of adequately describing the dynamics of exchange prices in real time is not available in literature, as far as we know (see, for example, reviews in [Ippoliti and Cheng, 2017]). It is our purpose in this paper to develop such a dynamic economic theory, and fulfillment of such purpose, among other things, will help to confirm (or refute) the foundations of probabilistic economic theory, which is of particular interest to us. Looking ahead, we note that here, we also found the same "striking simplicity of movement" of market agents, despite the fact that the exchange, without any doubt, is a complex dynamic nonequilibrium probabilistic system.

We have selected several different assets traded on the Moscow Exchange as the initial objects of our study, for the simple reason that historical data on the trading of these assets have recently been posted on the exchange website, so any researcher can use them to verify the correctness of our calculations. Let us emphasize that these historical data contain almost all quotations of all exchange agents at each moment of time during the whole trading day, not only quotations in a small «cup» near the current market price.

Let us clarify once again: this book represents, in fact, the results of the subsequent development of probabilistic economics, which we developed earlier [Kondratenko, 2005, 2015] for quantitative research of multicommodity multi-agent market economies. This theory in a fairly general form is developed on the basis of one axiom and six general principles that have such a simple and clear logic and rationale under them that they can be considered mandatory for inclusion in any sufficiently adequate economic theory. The way this theory can be used to set and quantitatively solve problems of real economic practice is illustrated in the mentioned works by examples of the simplest model economies, namely one-commodity economies with one buyer and one seller. In the present paper, we show how these theories can be used to quantitatively describe more complex real market economic systems, e.g., highly organized regulated commodity, stock or financial markets with a generally unlimited number of agents, namely exchanges. It turned out that for this purpose it is enough to introduce one more important assumption, namely the equivalence hypothesis, which we will describe

further in detail. The good agreement of the calculated exchange prices and trade volumes with the experimental values during the whole day trading session for various assets serves as a direct proof that the theory of exchanges is based on correct principles and hypotheses. The main final result of this work can be considered to be the creation of the organized markets (first of all exchanges) theory fundamentals, the application of which already at the initial development stage has given us an opportunity to shed light on the basic rules governing the functioning of highly organized markets.

# Chapter 1

## FUNDAMENTALS OF PROBABILISTIC ECONOMICS

"In what follows I have endeavored to reduce the complex phenomena of human economic activity to the simplest elements that can still be subjected to accurate observation, to apply to these elements the measure corresponding to their nature, and constantly adhering to this measure, to investigate the manner in which the more complex economic phenomena evolve from their elements according to definite principles".

Carl Menger [2007]

### 1.1. PROBABILISTIC NATURE OF ECONOMIC SYSTEMS

This chapter describes probabilistic economic theory in sufficient detail, starting from the formulation of the most general statement of the problem to the derivation of the fundamental formulas, using the simplest model of a two-agent economy as an example. To begin with and to avoid misunderstandings and ambiguities, let us repeat once again that, according to the ideology of the physical method, probabilistic economics is a theory that is developed using formal methods of theoretical physics or, in other words, by analogy with how theoretical physics is developed, but, fundamentally, it is an economic theory rather than a physical one, since it studies the structure and dynamics of the economic world, where rules are not in any way directly related to physical laws that describe the structure and dynamics of the natural world. This is already clear from the fact that the subjects of the economic world are people and their actions in the processes of exchange of goods and services, whereas the subjects of the physical world are particles and fields, in particular atoms. And, to be definite, let us also emphasize that this new economic theory was based on the classical concept of supply and demand, that was reinterpreted in the style of modern probabilistic scientific thinking.

There is no doubt that the modern real economy is a complex, nonequilibrium, dynamic system. Therefore, it is possible and necessary to actively study its structure and dynamics in different ways and from different points of view. Our point of view is that we look at the economy mainly as a set of a huge number of intelligently thinking and dynamically acting people, each of whom is "not only *homo sapiens*, but no less than *homo agens*" [Mises, 2005]. In order to solve problems and achieve goals, these "homo agens" or, more precisely, market agents, under the influence of constantly changing life and business circumstances, are forced almost continuously to make new important decisions related to the purchase and sale of goods and services, production, marketing, logistics, personnel control, etc. Being rational, these people try to make those decisions that will bring the greatest benefit and return on the efforts made. Such rational decisions can only be made on the basis of sufficient information available regarding the factors affecting their interests and decisions. This is why people are constantly in the process of searching for and processing new market information that is important to them. But the real world is such that we never fully have sufficient and reliable information about things of interest to us, primarily because of time constraints. Moreover, due to our limited mental and technical capabilities, we are not always able to correctly process and interpret even the information that we have at the right time and in the right place.

It is our deep conviction that human nature, as well as the nature of market economic systems, is such that all our knowledge of markets is only approximate, so all market decisions can only be approximately correct and optimal. Moreover, in practice we are Explicitly or implicitly aware of this fact and take it into account in our decision-making and evaluation of its consequences. Strictly speaking, our market decisions can only be probabilistic in nature. And since, according to our view of the market economy, all economic processes and phenomena are exclusively the



result of the actions of all the economic agents, it inevitably follows from all of the above that all economic processes and phenomena in the market economy are also, to some extent, probabilistic in nature. Consequently, only one step remains to draw the fundamental conclusion that the market economy is not just a complex, dynamic, nonequilibrium system, but also a probabilistic one [von Neumann and Morgenstern, 1970; Kondratenko, 2005, 2015; Mises, 2005; Waltuch, 2008; Keynes, 1921; Farjoun, Machover, 1983; Ball, 2003]. Therefore, in order to present a sufficiently complete description of such complex probabilistic systems, the corresponding economic theory should also be largely probabilistic. For this, at least, it is necessary to incorporate the concept of uncertainty and probability into economic theory at a suitable mathematical level, i.e., to develop a dynamic probabilistic economic theory that is sufficiently adequate to economic reality. Our research is devoted mainly to achieving this goal.

In this paper, we further describe the basic mechanisms of supply and demand formation, as well as the mechanisms of price formation, trade volume formation, and equilibrium setting in markets. The emphasis is on a detailed description of the nature of these fundamental market concepts and definitions in the standard general scientific language of physics and mathematics, which will allow us to critically re-evaluate these concepts and definitions within the framework of probabilistic economics.

The main paradigm of probabilistic economics for us is the following statement already formulated above. All markets consist of people – buyers of some goods and simultaneously sellers of these goods. Everything the market does is done by these people, and it is the actions of all these people in the market that determine all the market results [Mises, 2005]. A similar claim was once made by Richard Feynman et al. [1978] regarding physical multiatomic systems. It is this circumstance that allows us to establish an analogy between multi-agent market economic systems and multiatom physical systems and to bring physical approaches and language into economic theory. Based on this main paradigm and with careful consideration of all the basic features of human activity, explored in detail by Mises [2005], we will define the following six principles of probabilistic economics, which are essentially general postulates:

**Principle 1.** Supply and demand principle;

**Principle 2.** Agent principle;

**Principle 3.** Institutional and environmental principle;

**Principle 4.** Dynamic and evolutionary principle;

**Principle 5.** Trade volume maximization principle;

**Principle 6.** Uncertainty and probability principle.

The demand and supply principle reflects the fact that almost all important effects and processes in the market are determined by the interaction of supply and demand in the market. The agent principle testifies to the unique driving role of interacting market agents and the significant contribution to this interaction of social cooperation between them in the modern market economy. The institutional and environmental principle expresses the fact that the interaction of agents with different social institutions, different stakeholders and the external natural environment should be taken into account together with the interaction between the agents. The dynamic and evolutionary principle reflects the fact that market behavior is to some extent deterministic in nature and therefore, generally, can be analyzed by means of some equations of motion that describe both stationary behavior and the evolution of markets. The trade volume maximization principle determines the direction of the generally fairly free market under the influence of market forces. The uncertainty and probability principle shows that all market processes and phenomena, including decision-making, are probabilistic in nature and thus help us to understand what mathematical apparatus we need to adequately describe the behavior of agents and the market as a whole in the circumstances of uncertainty.

In essence, these principles define the main acting forces in the market and the main features of the market structure, so when performing specific studies of markets in our physico-economic models, they should all be taken into account, if possible, simultaneously, since they all represent market effects of the same significance in terms of influence on the final result of market operation, namely on the price structure and trade volumes in the market at each moment. If, for example, a certain economic study does not take into account the influence of the state, it cannot claim to be an adequate description of the modern economic world in which the role of the state is paramount: the state can both accelerate and suppress economic activity of other market agents. And the role of the state in the modern economy is twofold: it can both set important rules and introduce new institutions, thereby influencing the strategies of market agents, and be an active and strong market agent itself.

We emphasize that in these principles, if we consider them one by one, we can almost always find their «roots» in separate works of the previously cited authors of classical economic theory: Adam Smith (theory of commodity exchange, etc.), Carl Menger (theory of subjective value, etc.) and Ludwig von Mises (principle of methodological individualism, concept of market process, dynamic interpretation of S&D law, etc.). Here they are brought together, formulated quite clearly and it is emphasized that they should be taken into account simultaneously in the development of any economic theory, claiming for an adequate description of the real economic world, in which living people, rather than fictional characters like *homo economicus* in imaginary markets with perfect competition and with a supposedly all-powerful market hand, formed only by agents representing people and business without regard to the role of the state. What is brand new here, are the dynamic and evolutionary principle with the idea of finding equations of motion for market agents, the trade volume maximization principle to establish trends in the movement of market prices and the uncertainty and probability principle with the idea of using probabilistic strategies that the agents use to put their quotations and, most importantly, the elaborated mathematical apparatus, which allows for extensive quantitative research of real economic systems, including organized markets.

## **1.2. DIRECT AND INVERSE PROBLEMS OF ECONOMIC THEORY**

To develop a probabilistic theory of the exchange, a physical method of economic research was used, the essence of which consists in using standard theoretical approaches of physics for modeling and calculating economic systems, with subsequent constant comparison of calculation results with experimental data to verify the used approaches, models and theory as a whole and for the purpose of establishing thereby the applicability limits of models and theory as a whole in those cases where their improvement and development is feasible. If the results of calculations contradict the experiment to a sufficient degree, the theory should be rejected without hesitation, and the process of theory creation should be started anew. For the sake of certainty and clarification of the theoretical problems studied in this book, we further call this option of the physical method application “a solution of the direct problem of the economic theory”, in which the results of economic activity (in this book, stock trading) are calculated based on some initial principles (*ab initio principales*), which are then compared with the corresponding experimental data (in our case – with the results of stock trading). We oppose this approach to solving, shall we say, the inverse problem of economic theory, in which by mathematical processing of experimental data one seeks to obtain information about the studied economic system. It should be said that we borrowed the terms “direct and inverse problem” from the theory of elementary particles scattering, which solves formally similar problems. It is known that solving the inverse problem in physics is very difficult and there is no reason to think that solving the inverse problem in economics will be easier. Here we reserve to developing the methods for solving the direct problem of economics as applied to stock exchanges, so all conclusions and discussions in this book refer only to the direct problem of economic theory, unless specifically stated. In order to avoid misunderstandings, we will repeatedly emphasize this aspect of this study.

It seems to us that one of the results of the extensive application of the physical method to the research of economic problems by solving the direct problem of economics will be the development of a new interdisciplinary science "physical economics", the birth of which is taking place right before our eyes. At present, physical economics is developing quite intensively in different directions, and one of these directions is probabilistic economic theory, the simplest, one may say, starting version of which is the probabilistic economics considered in this paper; and it is the latter that served as a basis for the probabilistic theory of stock exchanges. The main advantage of this approach is the availability of methods for solving the direct problem of economics, namely numerical calculations based on the first principles of such parameters and functions that can be directly compared with Experimental data, in our case – with the experimental results of real-time exchange trading.

According to the new physical method of thinking and research in economics, which we share and develop, in this monograph as well, the main requirement to such economic models, defining their main purpose, is the possibility and, to some extent, even art, with a few important axioms, concepts and principles to harmoniously, competently and simultaneously include them in the theory. The latter is very important, since, by the definition of the purpose, all concepts and principles play certain roles of comparable importance in the studied economy. And of course, constant verification of the assumptions made, concepts and the theory itself through numerical calculations and comparison with experiment data is necessary. It is this emphasis on experimental verification of theories that distinguishes the physical method from all other approaches, including neoclassical theory, which is currently mainstream in economics. In other words, the physical method of economic research presupposes a constant reliance on experiment, as it is usually the case in physics: numerical solutions of a direct problem are compared with the corresponding experimental data. We are fully confident that due to its naturalness and obviousness, in the nearest future the physical method will become the main method of economic research, will be called simply a scientific method of economic research, so the necessity to call it a physical method in economics will disappear due to irrelevance and unnecessary.

### **1.3. CONCEPTUAL MODELING OF ECONOMIC SYSTEMS IN *PQ*-SPACE**

It is well known that the method of conceptual modeling of economic systems has long been widely used in economic theory. For example, the first and most famous concepts of neoclassical economics are S&D concepts. It is due to them that neoclassical theory has made a significant contribution to economic science. It helped economists better understand the basic components of the economic world, and with the help of a graphical interpretation this knowledge became more accessible to those interested in these issues, in particular students. In Austrian economics, the so-called method of ideal or imaginary constructions rightfully occupies one of the central places [Mises, 2005].

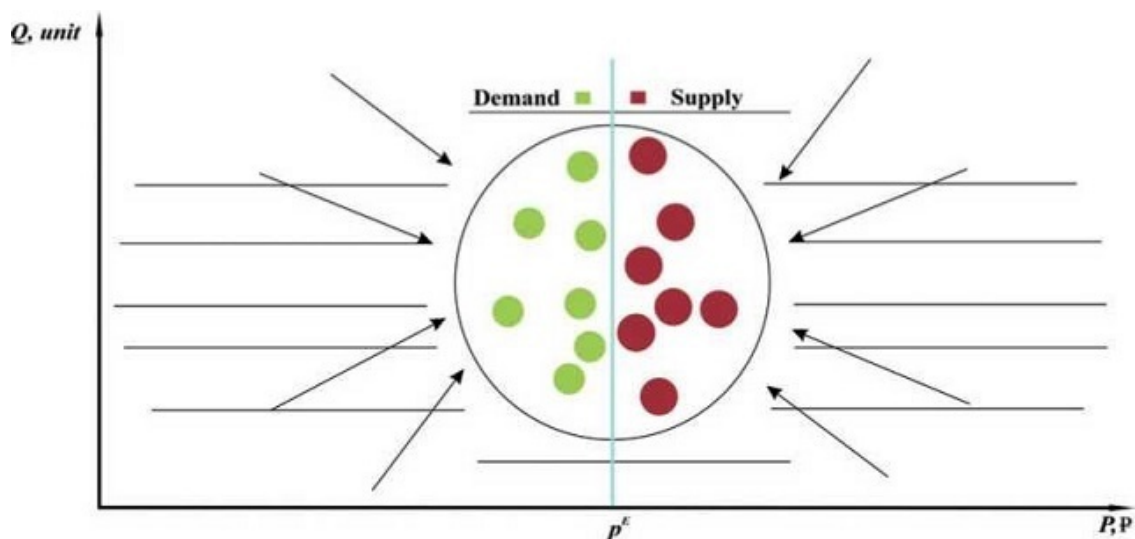
It is not customary in theoretical physics to emphasize the use of models, since theoretical physics itself can rightly be viewed as a conceptual mathematical modeling of physical systems. In particular, theoretical physics has developed the most advanced methods of theoretical modeling of complex systems. Moreover, here it has long been implicitly required from the researcher to perform quantitative numerical calculations of the structure and properties of such models with the highest possible accuracy, which, in turn, led to a striking development of quantitative methods in quantum mechanics of multiparticle systems, especially in quantum chemistry of multi-atom systems [Kondratenko and Neyman, 1990].

So, in this study deep structural and dynamic analogies between physical multi-atom systems and multi-agent economic systems are used in order to transfer conceptual, analytical and numerical methods from theoretical physics to theoretical economics. This transfer is performed by means of physical modeling of economic systems or, in short, physical-economic modeling. Our concept of

physical-economic modeling is based on the well-known fundamental ideas of classical economic theory, first of all of Austrian economics. They are then combined and eventually, with the help of additional concepts and mathematical body adopted from physics, transformed into a new economic theory. This combination is organized through formal approaches and methods borrowed from theoretical physics, starting from the introduction into economic theory of the formal economic space concepts, motion trajectories of market agents and the market as a whole in such space, and finishing with the substantiation of the principal possibility of using the method of equations of motion in economic theory. To avoid misunderstandings, we emphasize again that the role of theoretical physics here is only to provide the mechanisms for developing a probabilistic economics. Relying on this theory, step by step, taking into account all principles of the theory simultaneously, we create more complex physico-economic models, taking into account the experience with previous models, which will be repeatedly illustrated in figures and graphs.

Of course, all physical-economic models are essentially conceptual mathematical models, just as in physics. The notion of "physical modeling" is used to emphasize an analogy with the rules or principles of modeling in physics. Further, this term will become redundant and will not be used. In the course of developing conceptual models of economic systems we will consistently introduce the concepts and principles of our theory, which will serve as a basis for the models' frameworks, which, in turn, will be filled with new content step by step. We will start with constructing the simplest models with the help of analogies and formal methods of classical mechanics. For the sake of brevity, we will call such models classical models below. Naturally, only the first five principles will be used in the development of such a classical theory, or simply classics, since only they have analogues in classical mechanics.

So, Fig. 1.1 shows a typical graphic economic model of a market system, or simply, a market. This model, configured by analogy with models for physical multiparticle systems, uses a number of legends or conventions to demonstrate typical market structure.



*Fig. 1.1.* Graphical model of a single-commodity multiagent market economy in the economic two-dimensional price-quantity space. The dots inside the conventional sphere represent market agents: buyers (green dots) and sellers (red dots), forming demand and supply, respectively. The sphere is divided into two parts by the narrow blue line, which symbolically marks the narrow area of prices, where the transactions in the market are made at the current experimental price  $p^{Exp}$ . Buyers are in the left hemisphere and sellers are in the right hemisphere, since the buyers' prices are lower than the sellers' prices with very rare exceptions.



The main structural element of the model is the market itself, consisting of a certain number of interacting market agents: buyers and sellers. This market is not a closed system – it is an open system, because it is under the constant influence of its institutional and external environment, as well as other markets and other sources of influence. All these factors also serve as structural elements of the market, because they exert a strong influence on market agents, and without taking it into account it is impossible to obtain a reliable description of the mechanisms of market operation and its results.

Further, in order to be able to mathematically describe the dynamics of the economy, we should, just like in physics, place the entire market into some constructed economic spaces. Since such economic spaces, in contrast to the physical space, have an auxiliary and formal character, they can be constructed in different ways depending on the tasks to be solved. In this paper, it is appropriate to use the price-quantity space corresponding to two sets of independent variables, prices  $P$  and quantities  $Q$  for all traded goods on the market ( $PQ$ -space). For clarity, we denote the names of independent variables and their corresponding coordinate axes in bold. Despite its seeming simplicity, the concept of multidimensional economic space introduced in this study is of great importance in theory, since it provides a fundamental opportunity to describe the dynamics of economic systems in mathematical and graphical languages, as it has long been accepted in science.

This paper will extensively use the notion of "market structure", which includes both the agent structure of the market itself and all significant external factors and forces of various nature that affect the operation of the market. The study of the market structure and its various microstructures and the identification of the most important characteristics and connections between them represents the most important purpose of any economic theory.

The approach of probabilistic economics, aimed at solving the problem of adequate quantitative description of each agent's behavior in the market, as well as the behavior of the market as a whole, is based on one rather simple premise or hypothesis, which we will call an axiom. This axiom, which has a rather general character, forms the basis for the implementation of supply and demand concept in a probabilistic economy.

#### **1.4. AXIOM OF AGENT IDENTITY**

*All market agents are identical, only their supply and demand are different.* This axiom is the starting point in building up the theory. It says that in the context of studying the basic or determinant features of the behavior of market agents in the market and the market as a whole, especially in terms of the formation of market prices and trade volumes, all market agents have common or identical properties, depending mainly on the income and expenditure of agents, or, more precisely, on their S&D for the goods and services produced and traded in markets. In other words, all buyers with the same demand are identical, just as all sellers with the same supply are identical. It can also be said that such agents are indistinguishable from the point of view of influencing the outcome of market trading or exchange. This axiom is something similar to the principle of indistinguishability of particles in physics, but, naturally, it is not as strict as the principle of indistinguishability in physics. It is the S&D of agents that primarily determine their economic behavior in markets and, ultimately, the behavior of all markets; they are the only characteristics of agents and the main input data for calculation methods in probabilistic economics, i.e. the parameters that determine the studied economic system.

This axiom points to the possibility of constructing sufficiently generalized and accurate models of the agents' behavior in the market and, consequently, of the market as a whole on the basis of agents' supply and demand. It leads us to the right path for determining and defining general properties of the market agents' behavior, which, in turn, enables us to identify general regularities in the course of market processes. This gives us a reliable basis for building theoretical economic models at a fairly high scientific level, using formal physical and mathematical methods. We are convinced that

only these types of general market phenomena and processes fairly represent the main interest of any sufficiently accurate scientific investigation using the methods of theoretical and experimental economics. In other words, this axiom forces us to focus on building the economic theory as a sufficiently rigorous science, based on the study of the behavior of individual agents (see the principle of methodological individualism [Mises, 2005] in terms of their S&D, i.e., behavior determined by their individual demand or supply).

To avoid misunderstandings, let us make the following note. Of course, there are many aspects of the economic agents' behavior in markets, which are determined by specific nuances and peculiarities in the behavior of particular people and communities in different situations and in different markets, and which often cannot be described in terms of supply and demand. These nuances and peculiarities of specific economic agents in particular markets are important, of course, when studying the process of making specific market decisions in particular situations of planning their market intentions and strategies [Schiller, 2000], for example, when choosing their quotations in course of trading at an exchange, but they are not the subject of probabilistic economics in this study. Moreover, for a probabilistic economics, it is all these nuances and features that determine the supply and demand for each agent at any given time, and these S&D are the inputs to the probabilistic economics. But, again, neither these nuances and peculiarities, nor concepts such as expected utility and risk [Wickens, 2012], are the basic concepts or the subject of research in probabilistic economics. The same is true for the nuances and peculiarities of exchange markets: in probabilistic theory, there is no need to describe and explicitly account for them, unlike in traditional theories like efficient market theory [Fama, 1970]. After all, it is not all these specific nuances and peculiarities of decision-making processes of market agents, and not the properties of specific markets like perfect competition, etc., but the specific actions of market agents and their determining role in the operation, development and evolution of markets, and the economy as a whole, that constitute the main subject of research and the main content of economic theory, in our opinion. As we argued above, it is, of course, important to understand what market agents think, and why they make these particular decisions, and not others, but, nevertheless, this is the subject of research in other more applied disciplines of economic science.

## **1.5. PRINCIPLES OF PROBABILISTIC ECONOMICS**

As we know, specific definitions of supply and demand as well as formulations of the S&D concept and methods of building S&D functions differ from each other in each economic theory, depending on its goals, objectives and possibilities. For example, the concept of supply and demand in mainstream neoclassical economics is based on the intentions or action plans of market agents. The intentions or plans of market agents are a trade secret, but it is completely impossible to build up a quantitative economic theory on the basis of the unobservable intentions or plans of real market agents. On the contrary, probabilistic economics is built on the basis of real actions of these agents in markets that can be observed and measured. More precisely, in a probabilistic economics, market S&D are derived or result from real agent S&D, i.e. from real agent market actions. In this fundamental point, by the way, probabilistic economics is close to the Austrian school of economics, especially in the interpretation of Ludwig von Mises [2005]. In other words, unlike, say, neoclassics, the probabilistic functions S&D are determined on the basis of the real actions of market agents in the market. And they are calculated with strict account of the following six principles of probabilistic economic theory.

### **1.5.1. SUPPLY AND DEMAND PRINCIPLE**

Probabilistic economics is based on the most well-known concept of economic theory, namely the concept of S&D. In its most general form, this concept is formulated in probabilistic economics

as follows: all the main things that happen in the market depend on some specific balance of supply and demand, determined on the basis of decisions made and openly presented in the form of market orders or market quotations to buy or sell a certain amount of an asset at a certain price. And only what is determined in the market by supply and demand, expressed in this form, is the subject of the study of probabilistic economics. In this sense it can also be called S&D economic theory, and the S&D principle itself is the main element of its basis.

### **1.5.2. AGENT PRINCIPLE**

This is generally the most important concept, or paradigm, in respect of the markets. Here it is: every market consists of market agents, buyers and sellers, interacting quite intensively, and prone not only to competition, but also to mutually beneficial social cooperation. There are no other market forces in markets, except the forces of market agents' interaction. All market results are a consequence of the market agents' actions, even if their actions were strongly influenced by other factors: the state, institutions, etc. Everything that happens in markets is done by interacting market agents and therefore only agent-based models (agent action-based models or below simply action-based models) can provide a reasonable and reliable quantitative basis for any modern economic theory. And the actions of market agents in the market are exactly the issuing of bids or quotations to buy or sell, which was discussed above.

### **1.5.3. INSTITUTIONAL AND ENVIRONMENTAL PRINCIPLE**

Markets are never completely closed or free. All market agents are constantly influenced not only by other agents, but also by numerous external forces and factors. These external forces and factors, playing the role of boundary conditions, give economic systems harmony, integrity and stability. The most important of these are, of course, institutions of various kinds, such as the state, trade unions, laws, innovations, etc. Just as important might be such forces and factors of the external environment as other markets and economies, including foreign ones, as well as natural and man-made processes, etc. The influences exerted by each of these forces and factors on the structure of market prices and on market behavior can be comparable to the effect of the market agents' interaction. Moreover, the actions of strong external institutional and environmental factors can significantly both stimulate and impede effective operation of internal market mechanisms and even partially suppress effective functioning of the market as a whole. Thus, the influence of institutional and environmental factors should be adequately taken into account in models together with interaction between market agents.

### **1.5.4. DYNAMIC AND EVOLUTIONARY PRINCIPLE**

Modern markets are complex non-linear nonequilibrium dynamic systems, since all market agents are in constant interaction with each other and external forces, in other words, in constant motion in search of profitable connections to buy or sell goods and services. Buyers seek to buy as low as possible, and sellers want the highest possible price. Mathematically, we can describe this time-dependent dynamic and evolutionary market process as a movement in some formal economic space of market agents acting according to objective economic laws. Therefore, this movement has a somewhat deterministic character, and the market movement itself, or the evolution of the market system, in time can be approximated by equations of motion similar to the equations of motion in physics, such as the Lagrange equations in classical mechanics or Schrödinger equations in quantum mechanics.

### **1.5.5. TRADE VOLUME MAXIMIZATION PRINCIPLE**

In relatively free markets, buyers and sellers consciously and deliberately enter into trade deals with each other, because they enter into transactions only on mutually beneficial terms. It is not necessarily the case that they will seek to maximize their profits in every deal they make, since they understand that deals can only be mutually beneficial. But they try, usually, to increase their financial benefit by entering into as many of these mutually beneficial transactions as possible. Thus, it can be argued that the market as a whole seeks to maximize the trade volume, and moreover, to do it in monetary terms, as agents naturally seek, in the end, to increase their personal profits expressed in terms of money. This principle triggers the market process and the action of the laws of supply and demand. But the role of the state should also be taken into account, since it is present in the market as a full-fledged market agent and because, due to its enormous resources, it can, figuratively speaking, play not only the part of the first violin, but also the role of the conductor in this orchestra. Consequently, market dynamics can be approximated by applying the trade volume maximization principle to the market as a whole. Due to the fact that at present we do not have a developed mathematical body to use the more general principle of least action and the corresponding Lagrange equations of motion, the trade volume maximization still remains the only tool in this theory that helps to evaluate at least short-term trends in market dynamics.

### **1.5.6. UNCERTAINTY AND PROBABILITY PRINCIPLE**

Uncertainty and probability always accompany human action in the markets, therefore without taking them into account it is impossible to operate successfully for a sufficiently long period of time. This is caused by the very nature of human reasoning and the fundamental human inability to accurately predict the future state of markets. Moreover, according to the institutional and environmental principle, markets are constantly under the influence of various institutions and environmental factors. For these reasons, all market agent decisions and actions, and hence all market processes and phenomena, are essentially probabilistic, so they can only be adequately described by applying probabilistic mathematical methods and models. On the same grounds, demand and supply functions should also be described in terms of probability distributions.

To introduce some certainty, let us note that the first five principles, along with the axiom, constitute the basis for building a market model in which probabilistic effects are absent or not explicitly considered. We will call such a model classical because of its certain similarity to concepts of both classical economic theory and classical mechanics. Adding the sixth principle (the uncertainty and probability principle) to this base provides a basis for building a full-fledged probabilistic theory, which is naturally broader in nature and has much greater opportunities for further development. Obviously, from the methodological point of view, the classical model can be considered as a fairly good initial approximation of the probabilistic theory, which makes it possible to quantify the probabilistic effects by comparing the results of classical and probabilistic calculations. As a matter of fact, this exact possibility is the justification for building and using the classical model.

We believe that these six general principles can serve as a good basis to describe fully and adequately enough the main structural and dynamic properties of market economic systems and market processes in them. These principles and their rationale will be further discussed in more detail on several occasions. And the accuracy of the probabilistic economics built in this way and the limits of its applicability should and will, naturally, be determined in this study by comparing the results of calculations carried out using the numerical methods of this theory with the corresponding experimental data.



And now we will make three principal assertions, the validity of which is practically obvious from the very method of building a probabilistic economics.

**Statement 1.** Probabilistic economics can be regarded as a kind of unified economic theory focused primarily on the quantitative description of organized markets. Indeed, this theory is based on principles which are, to a certain extent, similar to concepts borrowed from various economic theories. For example, the fundamental concept of neoclassical theory, namely the concept of supply and demand, is also used as a basis for probabilistic economics. The concept of the market process and the subjectivist principle of value is borrowed from the Austrian school of economics; the institutional principle from institutional economics [Hodgson, 1988]; the position on the continuous evolution of economic systems from evolutionary economics [Nelson, Winter, 1982], etc. The mathematical body of probabilistic economics, organized similar to the methods of theoretical physics of multiparticle systems, allows to include these well-known achievements of economic thought into this probabilistic theory in a harmonious and clear way. To be fair, it should be said that all the principles of probabilistic economics follow as a necessity both from the method of theory construction itself and from the attempt to create adequate equations of motion for action-based models of economic systems by analogy with the equations of motion for physical multiparticle systems. This is how these principles were first derived and formulated [Kondratenko, 2005, 2015]. Moreover, it is obvious that these principles, as well as the axiom of the theory, are formulated intentionally in a rather general way, which emphasizes the fact that they are not absolute and can and obviously will be changed and refined as the theory develops and its possibilities and applications Expand. Just as it happens during creation of any physical theory, a creation focused on continuous verification of the theory by comparing the calculations results with experimental data, if, of course, such experiments have been performed and their results are known, otherwise, the authors of the theory try to suggest such experiments..

**Statement 2.** Probabilistic economics is not just one of many models of specific economic systems, but a rather universal method of numerical description and research for any market economic systems, both local and global. After studying them using this method and identifying the main effects, processes and regularities in their functioning, it is possible to build various simple models of these systems. Nevertheless, in order to avoid misunderstandings, it should be noted that probabilistic economics is far from being a “theory of everything”. It is aimed at investigating, perhaps, the most important, but by no means all important burning questions of economic theory, namely, how the structure and behavior of the market as a whole follows from the individual actions of market agents, i.e. from their individual presentation of demands and supplies. Therefore, let us emphasize once again that everything described in this monograph and everything asserted therein covers only the direct problem of economics, unless it is specifically stipulated.

**Statement 3.** The proposed mathematical apparatus for describing the market dynamics is built on using orders or quotations of market agents; therefore, it automatically takes into account all the principles of theory, since market agents take into account all the information coming to the market at any given moment in time when choosing the quotations. In other words, they are under constant influence of all forces and influences acting on the market at a given moment: this includes the influence of other agents, assets and markets; as well as the effect of institutional and environmental factors, etc., which is reflected in regular changes in their quotations.

In the next two sections we will describe in detail the mathematical body of the probabilistic economy based on the actions of agents and illustrate its work on the example of a simple model market with one buyer, one seller, and one traded commodity. It will be shown that the most specific features and regularities in the behavior of markets are already evident in such a simple two-agent model. An extension of this theory to multi-agent markets with one traded commodity will be presented in subsequent chapters.

## 1.6. CLASSICAL THEORY OF THE TWO-AGENT MARKET

Note that since we neglect all probabilistic effects in classical theory, or classics, we do not consider the uncertainty and probability principle in classics, although it is clear that it plays an important role in probabilistic theory. It is hardly worth seriously discussing which of these two theories is better. As in the case of classical and quantum mechanics, it is preferable to talk about different applications of classical (in a certain sense deterministic) and probabilistic theories, as we will demonstrate more than once below. Let us remind you that the classical theory in this book refers simply to an initial approximation of the probabilistic theory in which the principle of uncertainty and probability are not explicitly taken into account.

Thus, we will thoroughly describe this approach to the study of the economy dynamics, or evolution, within the framework of the classical economy using the example of the simplest model, namely, a market with one buyer and one seller selling one commodity, such as grain. The economic space in this case is obviously two-dimensional.

### 1.6.1. DISCRETE STRATEGY OF MARKET AGENTS

Let's consider a typical situation in a market, which has a real potential buyer and seller of a certain good, say, grain. The buyer wants to buy goods in quantity  $q^D$  at price  $p^D$ , and the seller wants to sell goods in quantity  $q^S$  at price  $p^S$ . These four parameters fully characterize the state of the market in the classical economy at each point in time. It is commonplace in the market that both prices and quantities of buyer and seller do not coincide. Therefore, if they both insist on their bid and ask, respectively, there will obviously be no deal. The oldest, well-established mechanism for resolving such trade disputes over the years since the emergence of markets is that the buyer and seller enter into trade negotiations with the aim of getting them to agree to a sale and purchase deal on terms that suit both parties. Let us describe this negotiation process in mathematical language as follows. Let the functions  $p^D(t)$  and  $q^D(t)$  denote the price and quantity of goods desired and offered by the buyer for buying during negotiations with the seller at a certain time  $t$ . Similarly, let the functions  $p^S(t)$  and  $q^S(t)$  denote the price and quantity of the good desired and offered by the seller for sell during negotiations with the buyer in the market. In their meaning, the values of prices and quantities introduced above are the main content of agent proposals to buy or sell the goods. Below, for brevity, we will denote these desired and offered values as buyer's and seller's quotations. And such a line of agents' behavior in the market will be called a discrete or point strategy, since at each time  $t$  these quotes are represented by one point in two-dimensional space, for example, point  $A$  with coordinates  $p^D(t)$  and  $q^D(t)$  for the buyer and point  $B$  with coordinates  $p^S(t)$  and  $q^S(t)$  for the seller, as presented in Fig. 1.2.

These quotations are made, of course, taking into account all the circumstances affecting the market operation: institutions, etc. In our view, quotations made by market agents are the essence of the main market phenomenon of classical economic theory in the view of the Austrian economic school, namely the market process [Mises, 2005], consisting of specific acts of choice and actions of market agents, which ultimately lead buyers and sellers to the conclusion of purchase and sale transactions. Graphically, we can depict these quotations as trajectories of agents' movement in economic space (Fig. 1.3). In real market life, these quotations are discrete time functions, but, for the sake of simplicity, we will depict them graphically (just like the S&D functions) as continuous straight lines. Such an approximation does not lead in this case to a loss of generality, because these functions are intended only to illustrate the most general details of the market mechanism and the way

they are described (see Fig. 1.3). In their economic sense, such diagrams characterize the temporal dynamics of the market.

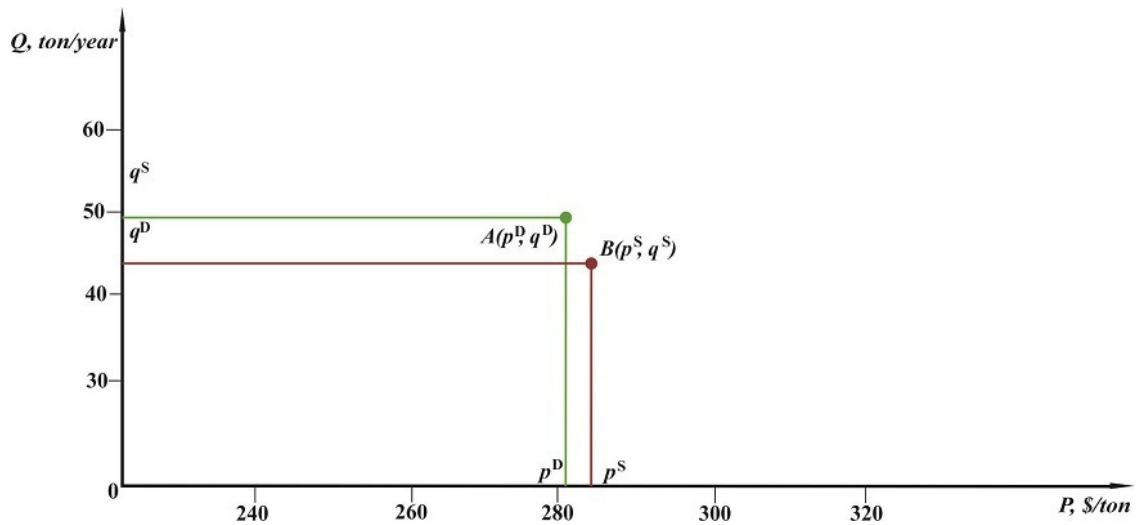


Fig. 1.2. Graphical representation of the discrete strategy of buyer's and seller's market behavior represented by the two points,  $A(p^D, q^D)$  and  $B(p^S, q^S)$ , in the two-dimensional price-quantity space at some particular moment in time for the model grain market.  $p^D = 280,0$  \$/ton,  $q^D = 50,0$  ton/year,  $p^S = 285,0$  \$/ton,  $q^S = 52,0$  ton/year.

We will speak (for the sake of brevity) of this aggregate agents' movement as market behavior, and sometimes as the economy evolution over time. All these terms are essentially synonymous in this context of discussion. Thus, by putting up desired prices and quantities as their quotations, buyers and sellers take part in the market process, proceeding here in the format of negotiations between bargaining people (*homo negotians*) seeking to bargain for the best terms for themselves in concluding a deal and achieving market goals. Let us note that in reality the actions of market agents include the procedures of concluding final deals along with quotations, but these procedures are automatically accounted by means of changing quotations by market agents after the conclusion of deals. Therefore, there is no need to Explicitly include the procedure of concluding transactions in the structure of agents' actions, it is enough to take into account only the quotation process in the course of trading.

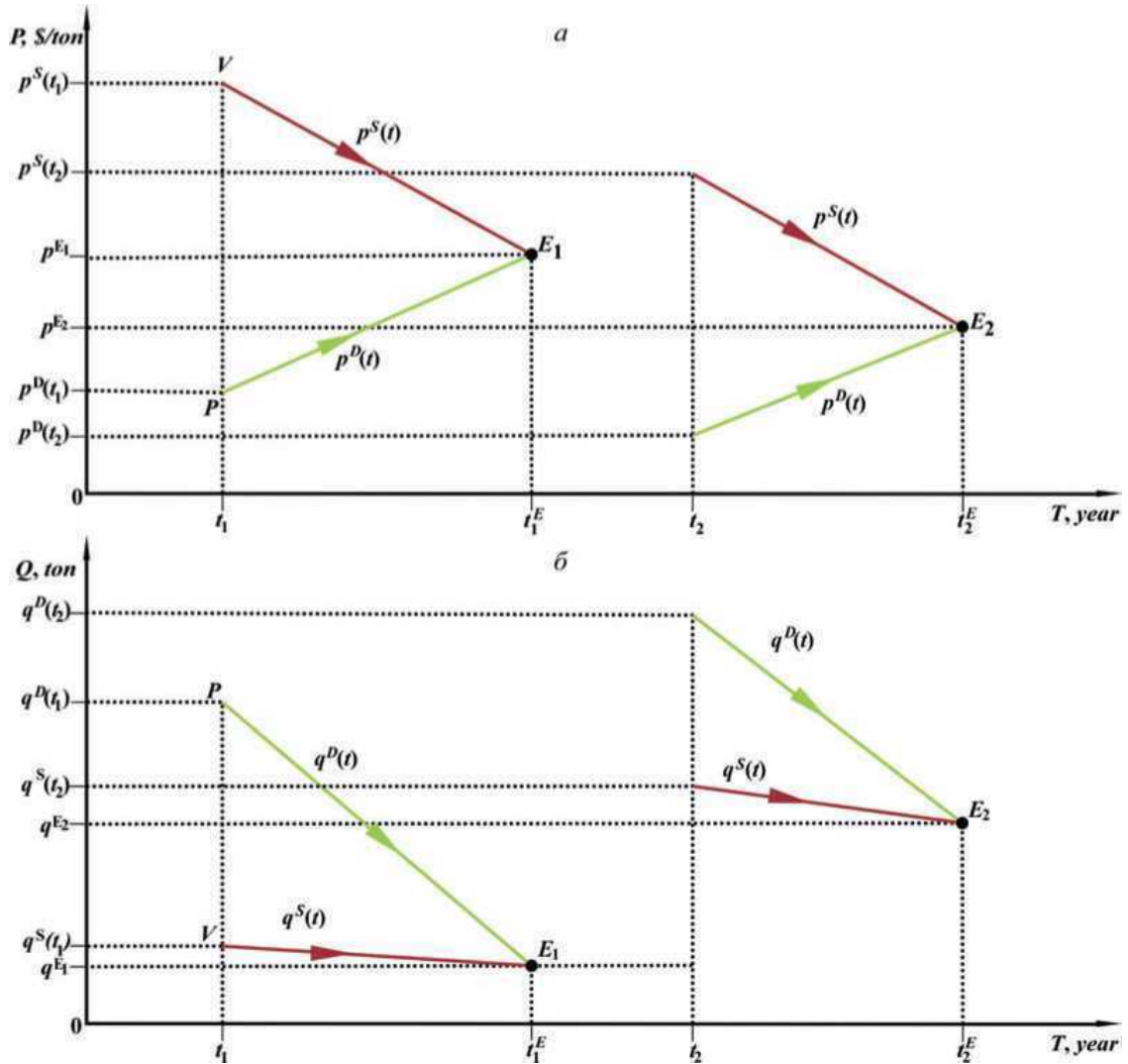


Fig. 1.3. Diagram of buyer and seller trajectories. The dynamics of the classical two-agent market economy in the economic space of price (a) and quantity (b) is depicted. Together, both parts of the figure represent the evolution of the economy over time in two-dimensional  $PQ$ -space.

This whole trading process, or simply bargaining, can be interpreted as a dynamic business exchange game between buyer and seller with the purpose of making a profit or achieving some other goal.

### 1.6.2. CONCEPT OF ECONOMIC EQUILIBRIUM IN CLASSICS

Let's suppose that the negotiations went well and ended with the conclusion of a sale and purchase deal at time  $t_1^E$ . This means that at that moment in time the values of prices ( $p^D(t)$  and ( $p^S(t)$ ) and quantities ( $q^D(t)$  and ( $q^S(t)$ ) in the quotations become equal, since obviously only specific mutually agreed price  $p_1^E$  and quantity  $q_1^E$  of goods can be specified in the contract. Let us assume that in this bargaining model it makes some sense to call these price and quantity values the market price and quantity of the commodity and to assume that the market itself comes to or reaches its equilibrium state at these price and quantity values. Formally, this is described using the following equations for the market price and quantity:

$$p_1^E = p^D(t_1^E) = p^S(t_1^E), \quad (1.1)$$

$$q_1^E = q^D(t_1^E) = q^S(t_1^E). \quad (1.2)$$

So, for the two-agent model we obtained this trivial but significant result: the very fact of reaching equilibrium makes it possible to conduct a transaction and maximize the volume of trades in monetary terms. In this simple case the conclusion is quite obvious: no agreement, no equilibrium, no transaction, trading volume is zero. But we will further show that this conclusion has a rather universal nature, which agrees with the postulated trade volume maximization principle. By the way, it is easy to show that in the framework of the neoclassical theory the maximum trade volume in natural terms, i.e. the maximum quantity of traded goods is reached at the equilibrium point.

Further on, since life does not stand still, the buyer and the seller can meet again and make new deals, but under new conditions and, obviously, with other prices and quantities, then for convenience we will call  $p_1^E$  the first market price, and  $q_1^E$  the first market quantity. Thus, at time  $t_1^E$ , the interests of the buyer and the seller have coincided for the first time, and they have been optimally satisfied by concluding a sale-purchase transaction. In this case agents, naturally, in the course of the market process (negotiations and changes in quotations) implicitly took into account the influence of the external environment and institutional factors on this and other markets, i.e. the economy as a whole. Here one can notice a similarity in the motion of the economic system in economic space, described by the trajectories of the buyer  $p^D(t)$  and  $q^D(t)$  and the seller  $p^S(t)$  and  $q^S(t)$ , and in the motion of the two-particle physical system in real space, described by the trajectories of particles  $x_1(t)$  и  $x_2(t)$ , which, by the way, are also the result of a certain physical maximization principle, namely, the principle of least action on the physical system.

The noted analogy with the physical system suggests using a similar mathematical body, analytical and graphical. In Fig. 1.3, to begin with, we provided a graphical representation of these trajectories of agents' motion as a time function using suitable coordinate systems time-price ( $T, P$ ) and time-quantity ( $T, Q$ ), similar to the construction of particle trajectories in classical mechanics. Please, note that figure 1.3 reflects a certain standard situation in the market, when buyer and seller intentionally meet at a point in time and start discussing a potential deal by mutual exchange of information about their conditions, first of all, desired prices and quantities of goods. During the negotiation, they continuously change their quotations until they agree to final terms on price  $p_1^E$  and quantity  $q_1^E$  at the time  $t_1^E$ . Such a simple «negotiation» market model is applicable, for example, to the economy of a fictional island on which, let us say for certainty, once a year there is a negotiated grain trade between a farmer and a hunter. To introduce some certainty, let us assume that they use the American dollar, \$, for settlement. For clarity, in Fig. 1.3, as in the following figures, we use arrows to reflect the direction of agents' movement during the market process.

So, in our classical negotiation model, up to the moment  $t_1$  the market is in the simplest dormant state, there is no trade at all. At time  $t_1$ , a buyer and a seller of grain appear in the market, and set their initial desired prices and quantity of grain:  $p^D(t_1)$ ,  $p^S(t_1)$  and  $q^D(t_1)$ ,  $q^S(t_1)$  points  $P$  and  $V$  in the graph show the position of the buyer and the seller at the initial moment of time  $t_1$  when trade negotiations begin. Naturally, the desires of buyer and seller do not immediately coincide; the buyer wants a low price, but the seller is fighting for a higher price. However, both need to reach an understanding and subsequent deal, otherwise the farmer and the hunter will have a difficult next year. The negotiation process continues, with the market agents' process of changing their quotations

reflecting its progress. As a result, the positions of the market agents converge, and they coincide at time  $t_I^E$ , which corresponds to the point of trajectories intersection  $E_I$  on the graphs.

On mutually beneficial terms at time  $t_I^E$  a voluntary transaction is performed. Then the market sinks back into a dormant state until the next harvest and the next year's sale at time  $t_2$ .

Let's assume, for certainty, that the new season's crop has increased, so  $q^S(t_2) > q^S(t_1)$ . In this case the seller obviously has to set the starting price lower right away,  $p^S(t_2) < p^S(t_1)$ , while the buyer also takes the opportunity to lower the price and increase his amount of grain:  $p^D(t_2) < p^D(t_1)$  and  $q^D(t_2) > q^D(t_1)$ . In this case it is natural to expect that the trajectories of the buyer and the seller will be slightly different, and the agreement between the buyer and the seller will be reached with different parameters than in the previous bidding round.

Conditionally, we will describe the situation in the market at each moment of time using a set of real market prices and quantities of real transactions that actually take place in the market. As can be seen from Fig. 1.3, in our model real transactions occur in the market only at such moments of time as  $t_I^E$  and  $t_2^E$  when the following market equilibrium conditions are true (points in Fig. 1.3)

$$p^D(t_i^E) = p^S(t_i^E) = p_i^E; \quad (1.3)$$

$$q^D(t_i^E) = q^S(t_i^E) = q_i^E; \quad (1.4)$$

$$D^0(t_i^E) = S^0(t_i^E) = TV(t_i^E), \quad i = 1, 2, \dots \quad (1.5)$$

### 1.6.3. CONCEPT OF SUPPLY AND DEMAND IN CLASSICS

In formulas (1.3)-(1.5) we used several new concepts and definitions that require some explanation. We will explain it in some detail, as it is important for understanding the subsequent presentation of the theory. First, the concept of S&D plays one of the central roles in modern economic theory. The same applies to probabilistic economic theory, which, as we said above, can to some extent be interpreted as a theory of supply and demand. Intuitively, on a qualitative descriptive level, all economists understand what this concept means. Difficulties and discrepancies appear only in practice when trying to give a mathematical interpretation of these concepts and to develop an adequate method for their calculation and measurement. For this purpose, various theories containing different mathematical models of S&D have been developed. These theories use various S&D functions to formally define and quantify S&D.

In this paper we will also repeatedly provide various mathematical representations of this concept within the framework of probabilistic economics, complementing each other. For example, in the framework of our two-agent classical economics (negotiation model) let's represent the S&D functions as follows:

$$D^0(t) = p^D(t) \times q^D(t); \quad (1.6)$$

$$S^0(t) = p^S(t) \times q^S(t). \quad (1.7)$$



In equations (1.6) and (1.7), we have defined at each time  $t$  the total buyer demand function,  $D^0(t)$  and the total seller supply function,  $S^0(t)$ , as the multiplication of price and quantity quotations. For brevity, we shall hereafter refer to them simply as supply and demand functions, i.e., we shall omit the word «total» unless this could lead to confusion. These functions can easily be depicted in the time and S&D coordinate system, namely:  $[T, S\&D]$ , as shown in Fig. 1.4, which shows a diagram of the complete S&D functions. As expected, the S&D functions also intersect at the equilibrium point  $E_I$ . More strictly, the equilibrium point is exactly the point in the diagram where the price and quantity quotations of the buyer and seller are equal. The fact that the S&D functions are also equal at this point is a simple consequence of their definition and the equality of prices and quantities in this point.

The last remark concerns the formula for estimating the market trade volume (Trade Volume, hereafter  $TV$ ) in the market  $TV(t_I^E)$  between a buyer and a seller at those moments in time when they come to a mutual understanding and conclude a transaction at an equilibrium point. Clearly, one can simply multiply the equilibrium values of price and quantity in this classical market model to obtain the trade turnover, or the total volume of all transactions, which follows from the above formula. The dimension of trade volume is the product of the dimensions of price and quantity; in our example, it is \$. The same is true for the dimensions of the functions S&D, namely  $D^0(t)$  и  $S^0(t)$ . Based on Fig. 1.4 we can conclude that it is at the equilibrium point that the trade volume reaches its maximum value. This result, which is self-evident and trivial in this case, is, in our opinion, rather general and principled: using it, we can deduce an assumption that markets tend to reach the equilibrium where maximum sales in monetary terms are achieved. It is possible to formulate this statement differently – in the form of the following hypothesis: markets strive for the maximum trading volume that is reached in equilibrium conditions, which agrees with the principle of trade volume maximization.

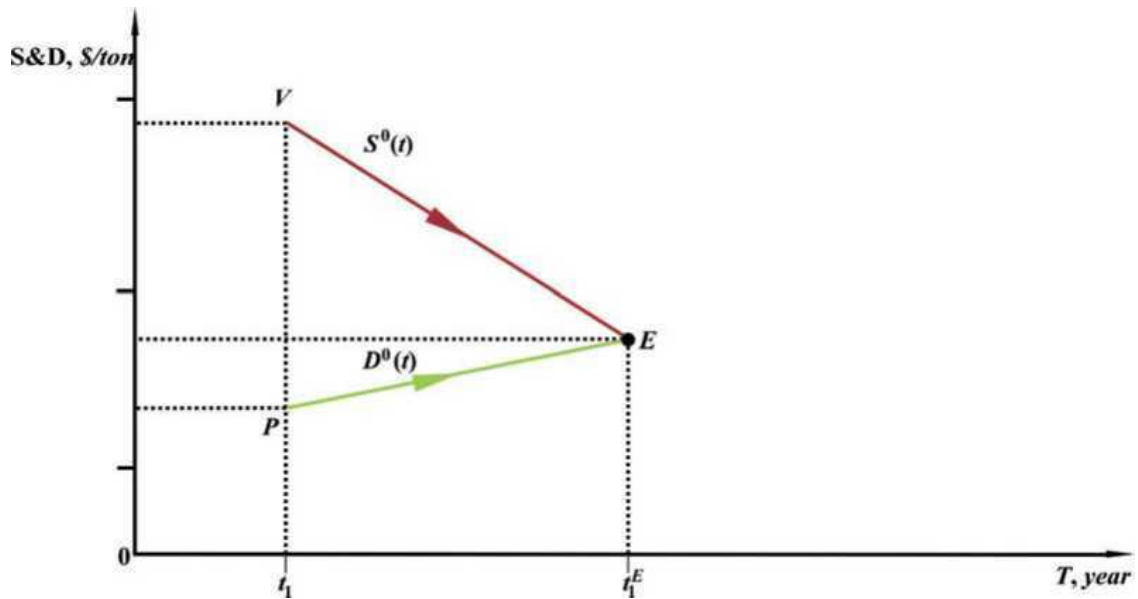


Fig. 1.4. Diagram of functions S&D, reflecting the dynamics of the classical two-agent market economy in the coordinate system  $[T, S \& D]$  in the first time interval  $[t_1, t_1^E]$ .

Further, similarly to classical mechanics, we can consider prices and quantities of market agents as trajectories of market agents in two-dimensional economic space using the coordinate system  $[P, Q]$  as shown in Fig. 1.5. Let us clarify that time  $t$  in this parametric representation of functions S&D is an implicit parameter. Generally, this parametric representation gives nothing new compared to Figs. 1.3 and 1.4. Nevertheless, there is one interesting nuance here – the similarity of this diagram

with the traditional picture in the neoclassical S&D model, namely the Marshall cross. We will touch on this issue a little later, but for now let's look at some of the features in Fig. 1.5. First, as the arrows show, the buyer and the seller move toward each other in terms of price: the seller lowers it, and the buyer, on the contrary, raises it. Thus the figure reflects normal market negotiation processes. Second, usually during negotiations, quantity quotations are reduced by both agents, i.e. both the buyer and the seller.

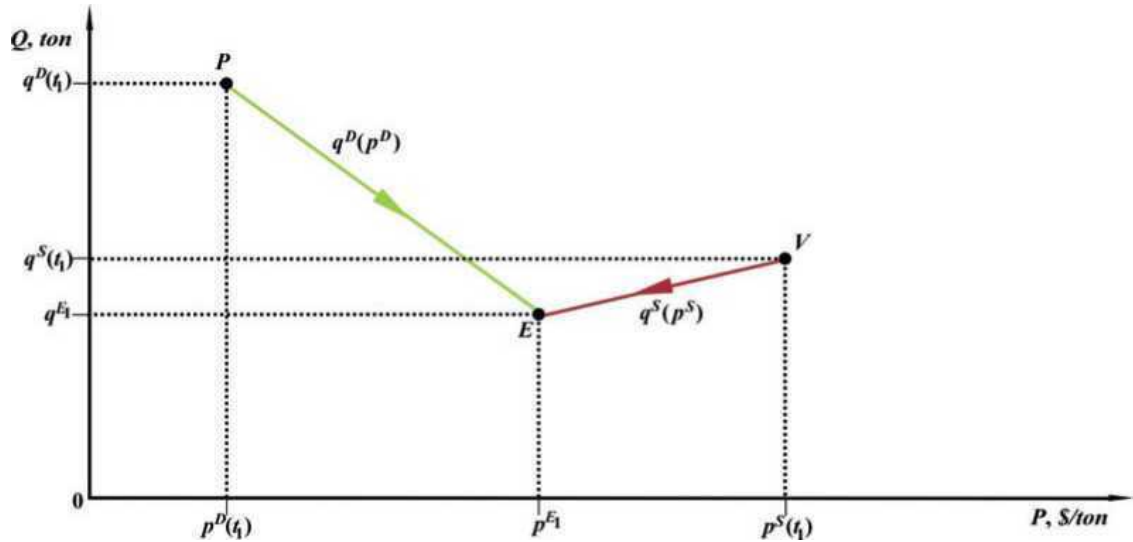


Fig. 1.5. Dynamics of the classical two-agent market economy in the two-dimensional economic space of price-quantity in the first time interval  $[t_I, t_I^E]$ .

Clearly, all agents want to buy or sell less goods at a compromise market price than at the desired prices they stated at the beginning of the trade. These factors together determine that the slope of the demand curve  $q^D(p^D)$  is negative and the slope of the supply curve  $q^S(p^S)$  is positive, just as the S&D functions in the neoclassical model «should» be. But this visual similarity is incomplete, because the economic meaning of these pictures in the two theories differs significantly: in the classical model it is a description of the actual process of negotiations in order to reach a deal, and in the neoclassical model it is a description of strategies of behavior of agents in the market in terms of neoclassical supply and demand curves,  $q^D(p^D)$  and  $q^S(p^S)$ . We emphasize that while in neoclassics these curves, by definition, represent as it were the actual functions of supply and demand, in classics these curves are simply a graphical representation of the price and quantitative time trajectories in the form of one trajectory in the course of trading. Thus, the classic economic theory does not assume the existence of any definite dependence of the agents' quantitative quotations on price quotations, i.e. the existence of any definite functions  $q^D(p^D)$  and  $q^S(p^S)$ .

In conclusion, we would like to emphasize that, as we have seen, if agents insist on their initial offers and show no willingness to bargain and compromise, the volume of bargaining will be zero. It is the willingness of the buyer and seller to modify their initial offers that leads to bargains. Thus, we can argue that market agents should initially include into their strategies a certain possible range of prices and quantities for their quotations. From this point, only one important step remains to build a better probabilistic model.

## 1.7. PROBABILISTIC THEORY OF THE TWO-AGENT MARKET

In order to achieve greater transparency of the presentation, we will also reserve ourselves in this section to describing the details of probabilistic theory on the example of the two-agent model of the grain market.

### 1.7.1. PROBABILISTIC STRATEGY OF MARKET AGENTS AND CONCEPT OF SUPPLY AND DEMAND IN PROBABILISTICS

We have come to the most intriguing point in the presentation of probabilistic economics, namely, we will now include the sixth principle – uncertainty and probability – to the theory. We will proceed as follows: first, for the analogy with theoretical physics, or more precisely, with the procedure of transition from classical mechanics to quantum mechanics to be clearly visible, and, second, we will try not to lose key aspects of describing the economic character, i.e. meaningful and rational behavior of agents in the market. The latter concerns, first of all, the process of agents' decision-making about the strategy of behavior in the market as well as the method of mathematical representation of market actions implementing these agents' decisions. Obviously, taking into account the principle of uncertainty and probability should in one way or another lead us from a point strategy of agents to some continuous strategy. Mathematically we will make this transition in exactly the same way as in theoretical physics we make the transition from temporal trajectories of particles to probability distributions of particles in space. Namely, let us move from a description of economic dynamics in classical economic theory in terms of temporal price and quantity agent trajectories,  $p^D(t)$ ,  $q^D(t)$ , etc., to a description of dynamics in probabilistics using continuous agent distributions of price and quantity probabilities  $D(p, q)$  и  $S(p, q)$ , which we will call probabilistic agent functions S&D. For certainty, let us note that these distributions themselves depend on trajectories,  $p^D(t)$ ,  $q^D(t)$ , etc., so they are themselves functions implicitly dependent on time. In order not to «obfuscate» the formulas by specifying this time dependence everywhere, we will often omit the time variable  $t$  in the formulas.

Let us briefly elaborate again on the rationale for this approach to physical-economic modeling. Market agents, forced to constantly work on the market in a continuously changing situation, are aware that the prices and quantities declared by them in the point strategy may not suit the counterparty at a given time. Based on previous experience in the market, they are well aware that they cannot know exactly how prices and quantities will develop in the market even in the near future. They are already used to operating in a market with great uncertainty, entailing high risks and the resulting potential costs. As a result, they realize that in the market they should always consider all their decisions and actions as possible with a certain degree of probability. This probabilistic aspect of the process of market decision-making is of great importance for understanding the behavior of agents in the market and the market as a whole [Mises, 2005; Gilboa et al., 2008]. Market agents think and act as *homo oscillans*. That is why they are forced to enter the real market not with discrete strategies, but with continuous strategies which can be represented by continuous probability distributions with certain widths correlated with the amount of uncertainty in the market situation in a given period of time.

### 1.7.2. PQ-FACTORIZATION OF AGENT S&D FUNCTIONS

As already noted, the description of continuous strategies requires the use of two-dimensional functions S&D,  $D(p, q)$ , and  $S(p, q)$ . Of course, it is rather tedious to calculate and analyze two-dimensional functions representing three-dimensional surfaces already in the case of a one-

commodity market. For this reason, we will take one more step in simplifying our models, which will make it possible to perform economic calculations of real multi-agent markets and to analyze the results obtained by our method at the highest scientific level. Thus, we *a priori* assume that we can factorize the agent functions S&D with a sufficient degree of accuracy, i.e. we can approximate their representation as a product of one-dimensional functions as follows:

$$D(p, q) \cong C^D \cdot d^P(p) \cdot d^Q(q); \quad (1.8)$$

$$S(p, q) \cong C^S \cdot s^P(p) \cdot s^Q(q). \quad (1.9)$$

This type of factorization and the corresponding approximation, in which the price and quantitative variables are separated, will be called *PQ*-factorization and *PQ*-approximation, respectively. Here  $d^P(p)$  and  $s^P(p)$  are one-dimensional price functions,  $d^Q(q)$  and  $s^Q(q)$  are one-dimensional quantity functions of S&D normalized by definition to 1:

$$\begin{aligned} \int_{-\infty}^{+\infty} d^P(p) dp &= 1; & \int_{-\infty}^{+\infty} s^P(p) dp &= 1; \\ \int_{-\infty}^{+\infty} d^Q(q) dq &= 1; & \int_{-\infty}^{+\infty} s^Q(q) dq &= 1. \end{aligned} \quad (1.10)$$

$C^D$  and  $C^S$  are simply normalization factors. They are derived from the condition of such a natural normalization selection of the agent functions S&D:

$$\int_{-\infty}^{+\infty} D(p, q) dp dq = D^0; \quad D^0 = p^D \cdot q^D; \quad C^D = D^0; \quad (1.11)$$

$$\int_{-\infty}^{+\infty} S(p, q) dp dq = S^0; \quad S^0 = p^S \cdot q^S; \quad C^S = S^0. \quad (1.12)$$

Here  $D^0$  and  $S^0$  are obviously total demand and complete supply of the buyer and seller, respectively. Below we will also omit the word «total» for the sake of brevity. It is easy to show that the agent S&D-functions,  $D(p, q)$  and  $S(p, q)$ , normalized in this way, are dimensionless functions. It is also obvious that in the point or discrete strategy described above, one-dimensional functions are represented by the so-called Dirac delta functions as follows:

$$\begin{aligned} d^P(p) &= \delta(p - p^D), & s^P(p) &= \delta(p - p^S); \\ d^Q(q) &= \delta(q - q^D), & s^Q(q) &= \delta(q - q^S). \end{aligned} \quad (1.13)$$

Keep in mind, that the special Dirac function by definition is zero everywhere except at the zero point, where it is equal to infinity, and its integral from minus infinity to plus infinity is 1. By the way, these functions can be applied to describe the probability functions of monopolist and monopsonist supply and demand in real markets.

Accounting for uncertainty in agents' strategies should obviously lead to «blurring» of these functions and turning them into continuous dome-shaped functions with maxima at the points  $p^D$ ,  $p^S$ ,  $q^D$  и  $q^S$  and agent widths  $\Gamma^{DP}$ ,  $\Gamma^{SP}$ ,  $\Gamma^{SQ}$  and  $\Gamma^{DQ}$  respectively. It seems reasonable, both from the economic and technical point of view, in the first approximation to use normal, or, simply, Gaussians distributions [Kondratenko, 2015]. Then the demand function has the following form in this approximation:

$$d^P(p) \cong g^{DP}(p) = \sqrt{w^{DP}/\pi} \cdot \exp\{-w^{DP}(p - p^D)^2\}, \quad (1.14)$$

$$d^Q(q) \cong g^{DQ}(q) = \sqrt{w^{DQ}/\pi} \cdot \exp\{-w^{DQ}(q - q^D)^2\}, \quad (1.15)$$

$$D(p, q) \cong D^0 \cdot g^{DP}(p) \cdot g^{DQ}(q), \quad (1.16)$$

where the parameters  $w^{DP}$  and  $w^{DQ}$  (agent frequency parameters below) are related to the agent widths as follows:

$$w^{DP} = -4\ln 0,5 / \Gamma^{DP^2} \cong 2,77 / \Gamma^{DP^2}; \quad w^{DQ} = -4\ln 0,5 / \Gamma^{DQ^2} \cong 2,77 / \Gamma^{DQ^2}. \quad (1.17)$$

Formulas for demand have a similar structure naturally:

$$S^P(p) \cong g^{SP}(p) = \sqrt{w^{SP}/\pi} \cdot \exp\{-w^{SP}(p - p^S)^2\}; \quad (1.18)$$

$$S^Q(q) \cong g^{SQ}(q) = \sqrt{w^{SQ}/\pi} \cdot \exp\{-w^{SQ}(q - q^S)^2\}; \quad (1.19)$$

$$S(p, q) \cong S^0 \cdot g^{SP}(p) \cdot g^{SQ}(q); \quad (1.20)$$

$$w^{SP} = -4\ln 0,5 / \Gamma^{SP^2} \cong 2,77 / \Gamma^{SP^2}; \quad w^{SQ} = -4\ln 0,5 / \Gamma^{SQ^2} \cong 2,77 / \Gamma^{SQ^2}. \quad (1.21)$$

To avoid misunderstandings, note that formulas (1.17) and (1.21) express the relationship known for Gaussians between their agent frequency parameters and the widths, more precisely, the widths of Gaussians at half-height. The numerical values of these widths are set or selected explicitly or implicitly by the agents themselves, just as prices and quantities are set by them. But, very importantly, unlike price and quantity quotations, the «quotations» of widths are not explicitly exhibited either in negotiations or in organized markets. The values of these widths may not even be accurately realized by the agents themselves; in this respect, agents may act purely intuitively, depending on the market situation.

Summing up the intermediate results, we can briefly say that in this version of the theory the buyer's probabilistic demand function is described by four parameters, the price  $p^D$ , the quantity  $q^D$  and two widths  $\Gamma^{DP}$  and  $\Gamma^{DQ}$ . The same statement is of course true for the seller's probabilistic supply function. It is these eight parameters that take into account all of the relevant market information that

the buyer and seller use before they put up quotations at any given time in the process of trading in the market. And let us emphasize for clarity that usually both buyers and sellers declare or announce publicly and unambiguously only their price and quantity quotations, leaving the information about their widths "behind the scenes".

### 1.7.3. GRAPHICAL REPRESENTATION OF AGENT S&D FUNCTIONS IN $PQ$ -SPACE

For our model grain market the probability functions S&D are presented graphically in Figs. 1.6–1.9.

Obviously, for a two-agent economy, all S&D market function surfaces have a simple smooth structure with one maximum. Of course, for more complex economies the structure of the surfaces will be much more complex.

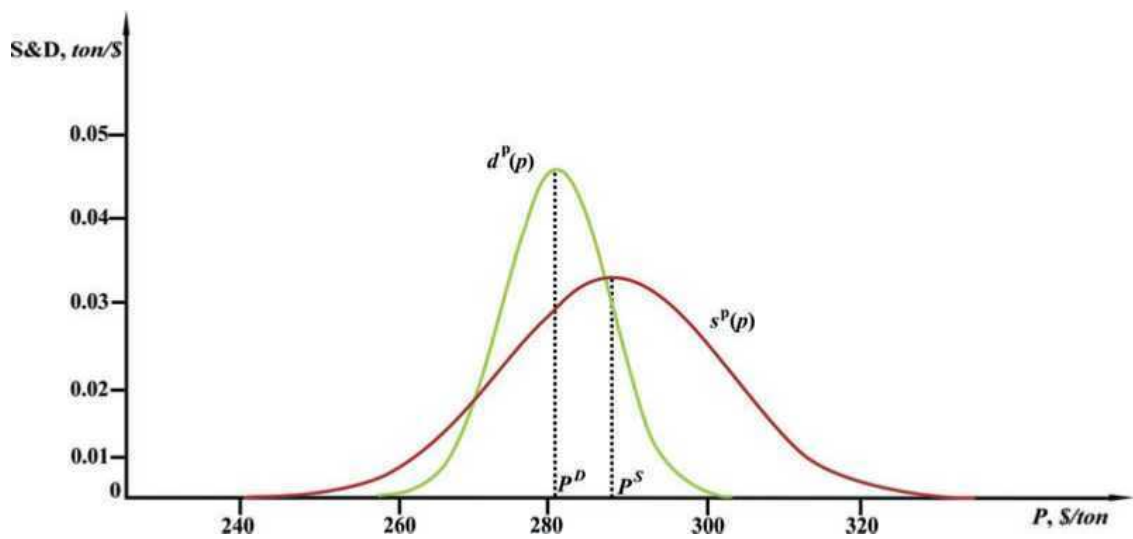
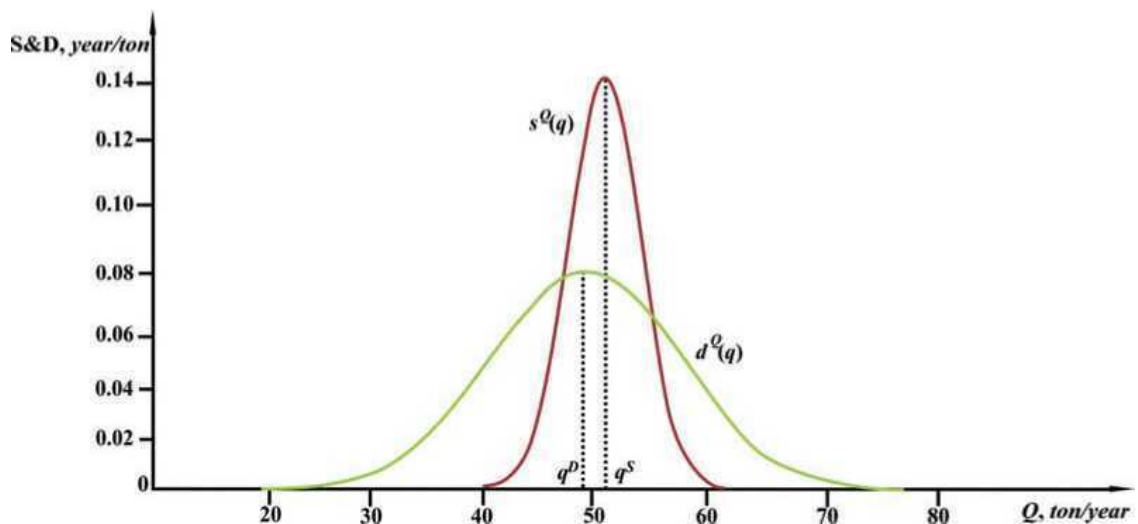


Fig. 1.6. Graphical representation in the rectangular two-dimensional coordinate system  $[P, S\&D]$  of one-dimensional price functions  $d^P(p)$  and  $s^P(p)$  as two-dimensional curves with maxima at prices  $p^D$  and  $p^S$  and widths  $\Gamma^{DP}$  and  $\Gamma^{SP}$  respectively. The values used for the widths are:  $\Gamma^{DP} = 23.8$   $\$/ton$ ,  $\Gamma^{SP} = 37.0$   $\$/ton$ .





*Fig. 1.7.* Graphical representation in the rectangular two-dimensional coordinate system [P, S&D] of one-dimensional quantity functions  $d^Q(q)$  and  $s^Q(q)$  as two-dimensional curves with maxima at quantities  $q^D$  and  $q^S$  and widths  $\Gamma^{DQ}$  and  $\Gamma^{SQ}$  respectively. The values used for the widths are:  $\Gamma^{DQ} = 26.4 \text{ ton / year}$ ,  $\Gamma^{SQ} = 6.8 \text{ ton/year}$ .

#### 1.7.4. PROBABILISTIC MECHANISM OF MARKET PRICING

Below we will discuss in detail all new concepts, main features and calculation details for our simplest two-agent system, so that we will not be distracted by their discussion in further consideration of more complex issues concerning the exchange. So, by definition and in its essence, the probabilistic function of demand  $D(p, q)$  (supply  $S(p, q)$ ) is the probability of the buyer (seller) concluding a deal to buy and sell the traded goods in quantity  $q$  at price  $p$ . If this is so, then, according to the standard concepts of probability theory, it is natural to define the probability of the transaction under these conditions as the multiplication of these probabilities:

$$F(p, q) = D(p, q) \times S(p, q). \quad (1.22)$$

We call this probability of making a deal a market deal function, and, for convenience, we also refer it to the market functions of supply and demand. Like the market functions  $D(p, q)$  and  $S(p, q)$ , it is dimensionless. For the sake of certainty, let us explain that, generally speaking, purchase and sale transactions can occur in the market at any time, at any price and in any quantity, within reasonable limits, but with varying degrees of probability. But if the transaction function is a sufficiently high and narrow bell with a single maximum with the parameters  $p^M$  and  $q^M$ , then almost all transactions will occur in the proximity of these values, so it is reasonable to consider these very values to be market prices and quantities. If the function of transactions looks otherwise, of course, these definitions are somewhat meaningless, and one should consider the mechanism of probabilistic pricing in detail. Below we will always assume that the function of transactions is such as to allow market prices and quantities to be determined in a fairly simple way. This is exactly the case we have graphically presented in Fig. 1.10 for our two-agent model of the grain market.

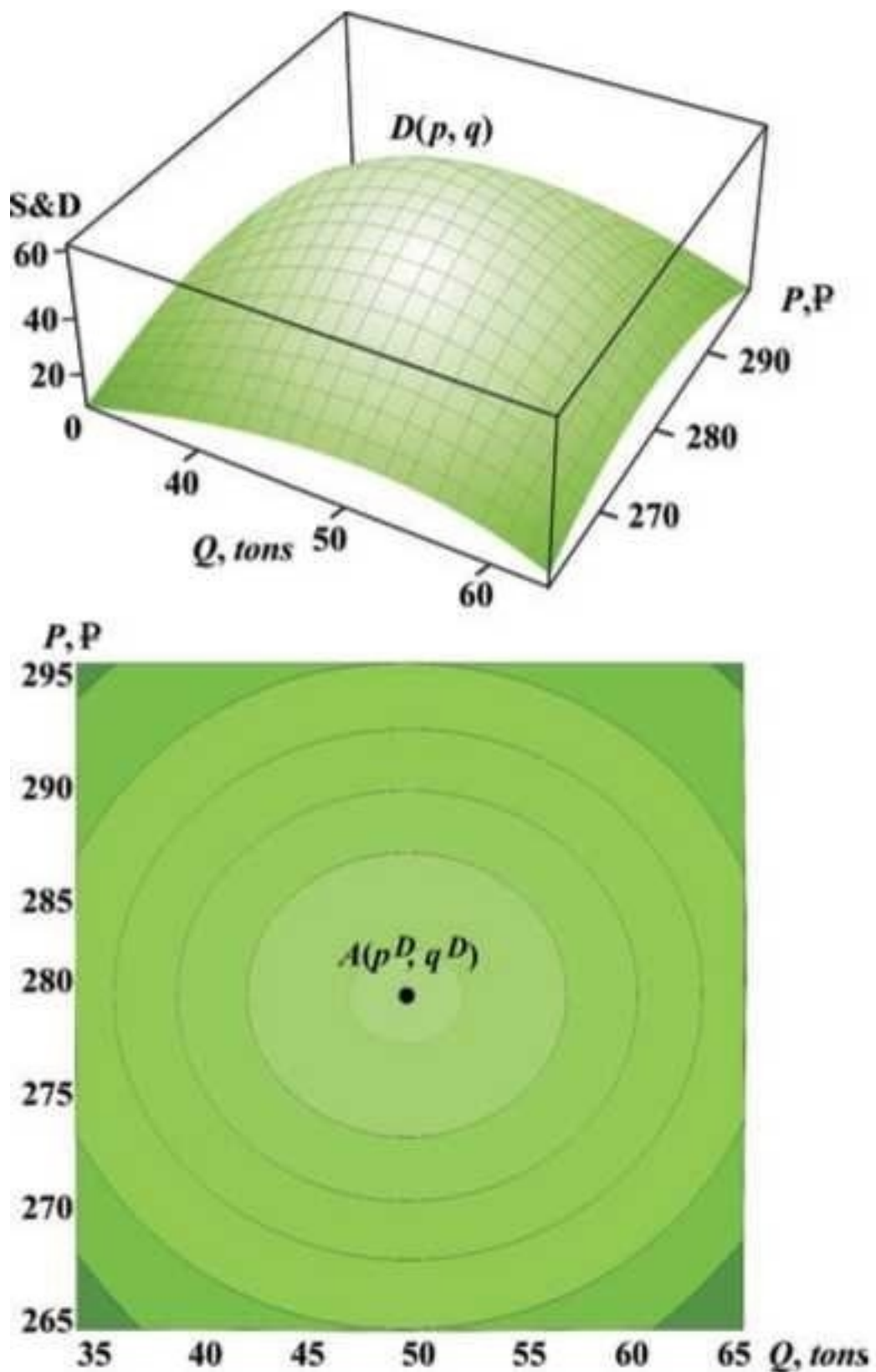


Fig. 1.8. Graphical representation in a rectangular three-dimensional coordinate system  $[P, Q, S\&D]$  of the two-dimensional buyer demand function as a three-dimensional surface  $D(p, q)$  with a maximum at the point  $A(p^D, q^D)$  in the plane  $(P, Q)$ .

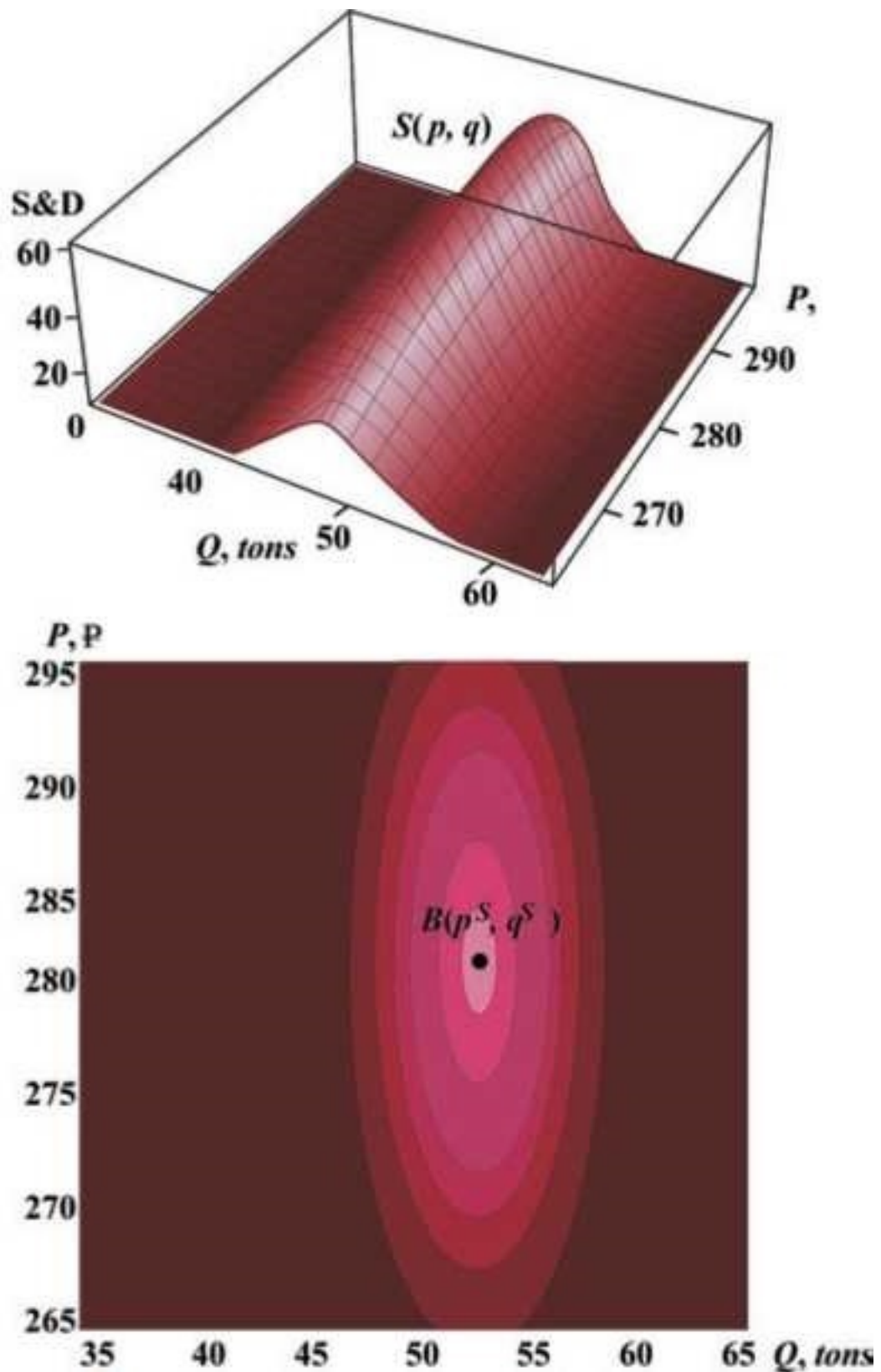
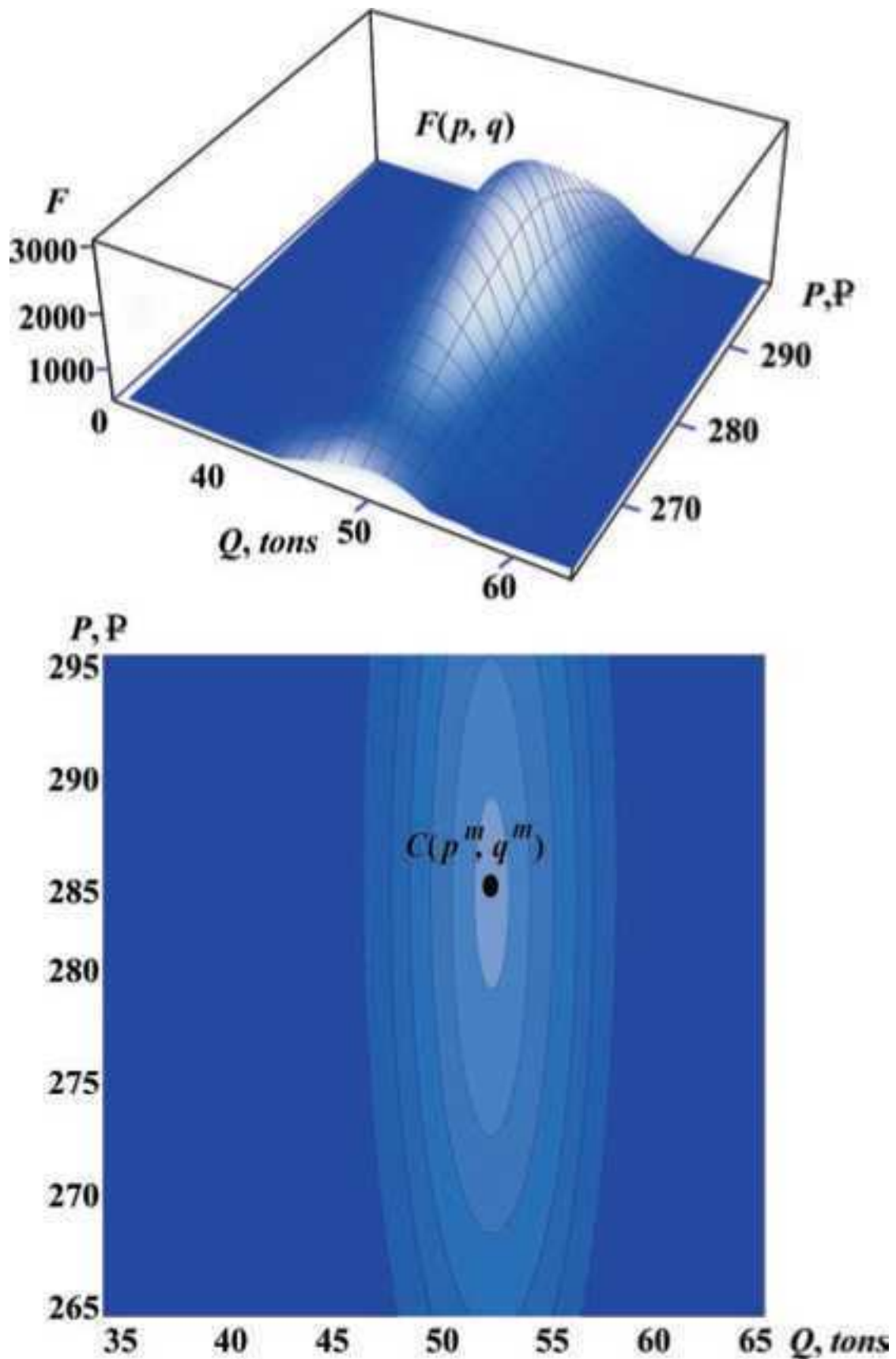


Fig. 1.9. Graphical representation in the rectangular three-dimensional coordinate system  $[P, Q, S\&D]$  of the two-dimensional seller's supply function as a three-dimensional surface  $S(p, q)$  with a maximum at point  $B(p^S, q^S)$  in the plane  $(P, Q)$ .

As expected, the surface of the market transaction function  $F(p, q)$  has only one maximum. For multi-agent economies, the structure can be much more complex.



*Fig. 1.10.* Three-dimensional graphical representation in a rectangular three-dimensional coordinate system  $[P, Q, F]$  of the three-dimensional deal surface  $F(p, q)$  in the form of a high and narrow bell with one maximum at the point  $C(p^M, q^M)$  in the plane  $(P, Q)$ . The graphical method of calculation gives the following results for market prices and quantities:  $p^M = 281.4$  \$/ton,  $q^M = 51.9$  ton/year.

Let us now turn to the question of calculating market prices and quantities within the framework of probabilistic economics. It is well known from the standard course of mathematical analysis that extrema of a multidimensional function should be defined as points on the corresponding surface in which the total differential of this function is 0. In our situation this condition leads to the following equation:

$$dF(p, q) = 0. \quad (1.23)$$

This equation is equivalent to the following two partial derivative equations:

$$\begin{aligned} \partial F(p, q) / \partial p &= 0; \\ \partial F(p, q) / \partial q &= 0. \end{aligned} \quad (1.24)$$

In terms of S&D functions, this system is transformed as follows:

$$\begin{aligned} \partial D(p, q) / \partial p \cdot S(p, q) + D(p, q) \cdot \partial S(p, q) / \partial p &= 0; \\ \partial D(p, q) / \partial q \cdot S(p, q) + D(p, q) \cdot \partial S(p, q) / \partial q &= 0. \end{aligned} \quad (1.25)$$

At this point it makes sense to introduce a new concept into theory, namely the concept of S&D market forces with such definitions:

$$\begin{aligned} U^{DP}(p, q) &\equiv \frac{\partial D(p, q) / \partial p}{D(p, q)}; & U^{DQ}(p, q) &\equiv \frac{\partial D(p, q) / \partial q}{D(p, q)}; \\ B^{SP}(p, q) &\equiv \frac{\partial S(p, q) / \partial p}{S(p, q)}; & B^{SQ}(p, q) &\equiv \frac{\partial S(p, q) / \partial q}{S(p, q)}. \end{aligned} \quad (1.26)$$

In terms of market forces we can write the system of equations (1.25) more compactly as follows:

$$\begin{aligned} U^{DP}(p, q) &= -B^{SP}(p, q); \\ U^{DQ}(p, q) &= -B^{SQ}(p, q). \end{aligned} \quad (1.27)$$

Obviously, this system of equations looks like a system of equality of S&D market forces at values of market prices and quantities. And it is similar to the system of forces equality at the static equilibrium point in classical mechanics. In other words, the system of economic equations (1.27) looks like a formulation of Newton's third law in classical mechanics. Substituting specific S&D functions from equations (1.16) and (1.20) into the system of equations (1.26), we obtain such simple and clear formulas for calculating market forces:

$$\begin{aligned} U^{DP}(p, q) &= -2w^{DP}(p - p^D); & U^{DQ}(p, q) &= -2w^{DQ}(q - q^D); \\ B^{SP}(p, q) &= -2w^{SP}(p - p^S); & B^{SQ}(p, q) &= -2w^{SQ}(q - q^S). \end{aligned} \quad (1.28)$$

As we can see, all forces have become one-dimensional functions in this model. Then, using these equations, we obtain a very elegant system of two independent linear equations to determine market prices and quantities:

$$\begin{aligned} -2w^{DP}(p - p^D) &= 2w^{SP}(p - p^S); \\ -2w^{DQ}(q - q^D) &= 2w^{SQ}(q - q^S). \end{aligned} \quad (1.29)$$

This system is so simple that you don't even have to solve it in the usual sense to get a very nice looking solution for market prices and quantities:

$$\begin{aligned} p^M &= (w^{DP} \cdot p^D + w^{SP} \cdot p^S) / (w^{DP} + w^{SP}), \\ q^M &= (w^{DQ} \cdot q^D + w^{SQ} \cdot q^S) / (w^{DQ} + w^{SQ}). \end{aligned} \quad (1.30)$$

Thus, probabilistic market prices and quantities in a two-agent economy, when using factorized agent functions in the form of Gaussians, are determined by averaging the corresponding agent parameters, with the frequency parameters of the agents serving as weights in this averaging. The fundamental point here is that these two simple, and independent, algebraic formulas, which include only four buyer parameters ( $p^D$

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